Elementary Particle Physics
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Lecture 03

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Interactions with Matter: Ionization Loss

Energy Loss by Ionization (Ionization Loss):

- Is the energy loss of charged particles due to electromagnetic interactions with the atomic electrons in the material.
- The ionization loss \( dE/dx \) is described by the Bethe-Bloch formula and is a function of the ionizing particle’s momentum \( p \) and mass \( M \);
  or, equivalently, of \( \beta=p/E \) (\( E \) is total energy) and \( \gamma=E/M \).
- For **non-relativistic particles**, the ionization loss behaves as \( \beta^{-2} \), i.e. low velocity particles are strongly ionizing (hence the excessive damage from exposure to alpha radiation).
- For **relativistic particles**, ionization loss goes through a minimum (minimum ionizing particles) followed (depending on the polarizability of the medium which causes shielding) by a “relativistic rise”. This rise is a relativistic effect as the strength/range of the transverse electric field grows with \( \gamma \) of the particle.

\[
-\frac{1}{\rho} \frac{dE}{dx} = \frac{4\pi ZN_A Q^2 e^2}{Am_e \beta^2} \ln \left\{ \frac{2m_e \beta^2 \gamma^2}{\langle \hbar \omega \rangle} \left[ 1+2\gamma \frac{m_e}{M} + \frac{m_e^2}{M^2} \right] \right\}^{-1/2} - \frac{\beta^2 - \delta/2}{\rho} \text{ Bethe-Bloch}
\]

with \( 4\pi N_A \frac{e^4}{m_e} = 0.307 \text{ MeVcm}^2 \)
The Energy Distribution of Primary Electrons

The primary ionization process is a statistical process of individual interactions. The cross section \( \frac{d\sigma}{dE'} \) that an electron receives a kinetic energy between \( E' \) and \( E' + dE' \) is

\[
\frac{d\sigma}{dE'} \approx \frac{2\pi Q^2 e^2}{m_e \beta^2} \frac{1}{E'} \quad \Rightarrow \quad -\frac{dE}{dx} = \rho \frac{Z}{A} N_A \int_{E_{\text{min}}}^{E_{\text{max}}} E' \frac{d\sigma}{dE'} \, dE' = \rho \frac{Z}{A} N_A \frac{2\pi Q^2 e^2}{m_e \beta^2} \ln \frac{E_{\text{max}}}{E_{\text{min}}}
\]

which is the Rutherford cross section and ignores spin and is therefore an approximation. Thus, the number of electrons in the medium that acquire kinetic energy larger than a chosen energy \( E_0 \) is calculated as:

\[
\frac{dN(E>E_0)}{dx} = \rho \frac{Z}{A} N_A \int_{E_0}^{E_{\text{max}}} \frac{d\sigma}{dE'} \, dE' = \rho \frac{Z}{A} N_A \frac{2\pi Q^2 e^2}{m_e \beta^2} \left( \frac{1}{E_0} - \frac{1}{E_{\text{max}}} \right) \approx \rho \frac{Z}{A} N_A \frac{2\pi Q^2 e^2}{m_e \beta^2} \frac{1}{E_0} \quad \text{for} \quad E_0 \ll E_{\text{max}}
\]

i.e. relativistic protons (\( \beta \approx 1 \)) in Argon gas at STP produce about 6 electrons/cm with energy larger than 15.8 eV, the ionization potential of Ar; one electron per cm has a kinetic energy larger than 100 eV, and so on.

The energy loss distribution has a very large tail and is well described by the Landau distribution. For this reason, and also because the \( E_{\text{max}}(\beta \gamma) \) may not be a relevant limit when the (rare) most energetic electrons with \( E>E_{\text{cut}} \) escape the detector medium, the “most probable” energy loss is much more useful than the average energy loss measured by the Bethe-Bloch formula.
The Range of Primary Ionization Electrons

The range of low-energy (non-relativistic) particles can be calculated by integrating their (kinetic) energy loss up to their total incident energy $E$:

$$R(E) = \int_{E}^{0} \frac{1}{-dE'/dx} \, dE'$$

For electrons, however, the path is very quickly randomized because of their small mass. An empirical relationship for the range of electrons in Argon gas at STP is:

$$\rho R_e \approx 0.71 \left(\frac{E}{\text{MeV}}\right)^{1.72} \text{gcm}^{-2}.$$
Energy Loss by Bremsstrahlung Radiation

The dominant energy-loss mechanism at very large $\beta\gamma$ is bremsstrahlung (literally: “breaking radiation”). We found that:

$$-\frac{1}{\rho} \frac{dE}{dx} = \frac{N_A}{A} \frac{4 \ln \frac{183}{Z^{1/3}}}{Z^{1/3}} \int_{\nu_{\min}}^{\nu_{\max}} \nu' \frac{d\sigma}{d\nu'} d\nu' = \frac{N_A}{A} \frac{Z^2 \alpha^3}{m_e^2} 4 \ln \frac{183}{Z^{1/3}} \left( \nu_{\max} - \nu_{\min} \right)$$

where $\nu$ is the energy (or frequency) of the emitted photon

- Note that the cross section grows with $Z^2$: high-Z materials are very effective “radiators”

- Note the crucial factor $m_e^{-2}$ in the cross section; the bremsstrahlung cross section for a muon, the next lightest charged particle, is reduced by a factor $(m_e/m_\mu)^2 = (0.511/105)^2 \approx 4 \times 10^4$. Typically, electrons radiate already above 10 MeV; muons only above $\sim$400 GeV/c, protons above 30 TeV.

- Note that $\nu_{\max} \propto E_{\text{inc}}$, and the radiation loss grows with incident energy $E$:

where $X_0$ is the “radiation length”; e.g. $X_0$(Pb; $Z=82$, $A=207$) 5.6 mm.
Ionization vs. Radiation ...

Note that for a muon bremsstrahlung becomes the dominant source of energy loss in Cu for $\beta\gamma = p/m \geq 4000$, corresponding to $p \geq 400 \text{ GeV}$, which is rarely reached at current colliders. Electrons start radiating at much lower momentum: $(m_e/m_\mu)^2 \approx 200^{-2}$, i.e. at about 10 MeV.

In Figure 35 the energy loss for electrons (dividing out the factor $E$) is shown.

The energy at which the radiation starts to dominate is denoted as the “critical energy” $E_c$; below $E_c$ electrons lose energy by ionization, and above $E_c$ by bremsstrahlung. Rossi’s definition: the point where the energy equals the energy loss per radiation length due to ionization:

$$\int_{X_0} \left[ \frac{dE}{dx} \bigg|_{\text{ion}} \right] \, dx \equiv E_c$$

An approximate expression is $E_c = 610 \text{ MeV}/(Z+1.24)$. 

Figure 35. The specific $dE/dx$ of electrons and positrons in Pb vs. electron energy.

Figure 36. Definitions of critical energy of Cu.
Energy Loss for Energetic Photons

Photons lose energy by the photoelectric effect, the Compton effect, and – at high energy – by pair production. The photoelectric and Compton scattering cross sections fall with photon energy $E$ as $E^{-1}$, and for energies above 1 MeV, pair production (described by the “same” diagram as bremsstrahlung!), becomes the dominant energy-loss mechanism.

A full calculation, using the same Feynman diagram as for bremsstrahlung, yields a “pair conversion length”, about 30% larger than a radiation length. Photon cross sections on Carbon and Lead are shown in Figure 37. Note that the characteristic length is directly proportional to $\sigma$. One observes that at photon energy above 10 MeV the pair production cross section dominates and becomes constant.

Figure 37. Photon cross sections on Carbon and Lead vs. energy.
Tracking Detectors

ATLAS Inner Detector (Tracking; no disks shown)
Silicon Trackers

Silicon tracking consists of wafers with individual diodes, typically strips spaced by 50 μm and several mm/cm long, etched on them which are depleted by a reverse bias voltage. The depleted region carries no current because of the absence of charge carriers, until a ionizing particle crosses the depletion region and creates large numbers of electron-hole pairs. The charge carriers follow the electric field to the diode electrodes and create a small charge pulse which is amplified and measured. Silicon *strip* sensors are fast, relatively radiation tolerant, capable of very high resolution and able to operate in a dense track environment.

Pure silicon is an insulator with very few charge carriers: i.e. the conduction band is empty and separated from the fully filled valence band(s) by a “large” band gap (5.5/1.12/0.66 eV in Diamond/Si/Ge). Valence electrons may be excited into the conduction band (leaving a hole behind) by thermal excitation $\propto \exp(-E_{\text{gap}}/kT)$ or by ionizing radiation; it takes an average 3.6 eV in Si (13/2.9 eV in Diamond/Ge) to create an electron-hole pair. And, conductivity in pure Silicon increases strongly with temperature.
Silicon Trackers

The density of a Silicon is roughly $10^3$ times larger than a gas, and the energy to create an e-h-pair is about 8 times smaller than for e-ion creation in Argon, so the number of primary charges per cm in Silicon is about $10^4$ times that in Argon gas, i.e. about $10^5$/mm and no intrinsic amplification mechanism is needed. However, to use Silicon as a detector of ionizing radiation, the number of electrons created by the passage of a charged particle must significantly exceed the number of thermally created electrons. At room temperature, 300 K, there are $1.5 \times 10^{11}$ thermal electron-hole pairs/cm$^3$ in pure Silicon, which swamps any ionization signal. In contrast, diamond at STP would work as detector, albeit at 5 times lower signal size.

Doping of Silicon and creating a p-n diode creates a depletion region devoid of free carriers. The depletion layer may reach the full depth of the Silicon wafer (~300 μm) with reverse biasing.

Obviously, the tracking region (required radius 1-2 m) cannot be fully filled with silicon sensors because $X_0$(Si) = 9.36 cm, and the tracker is typically constructed from sensors (about 10 cm × 10 cm in size) organized in several “barrels” (in central region) and “disks” (forward directions). Silicon strip detectors measure best perpendicular to the strips (about $50/\sqrt{12}$ μm), and worst in the longitudinal coordinate (about $10/\sqrt{12}$ cm). Space points can be obtained with “stereo” layers, i.e. strips oriented at an angle with respect to the main strip direction. Pixel detectors, typically 50 μm × 300 μm in size, at the cost of very high channel count, are an alternate solution for obtaining unambiguous space points.
Why Pixels?
Pattern Recognition in ATLAS Inner Detector: \( H \to bb \)
Only half of all hits shown (0\(<\eta\)< 0.7, TRT hits for \( z > 0 \) barrel)

\( \Rightarrow \) Vertexing and b-tagging will be challenging, and need pixel detectors!

\( H \to bb \) event at zero luminosity (left), and at design luminosity (right)
The ATLAS Silicon Pixel Detector

The ATLAS Pixel Detector provides three high precision measurements as close to the interaction point as possible, and mostly determines the impact parameter resolution and the ability of the Inner Detector to find short lived particles such as B-Hadrons. The system consists of three barrels at average radii of ~5 cm, 9 cm, and 12 cm (1456 pixel modules), and three disks on each side, between radii of 9 and 15 cm (288 pixel modules).

Each pixel is \(50 \, \mu m \times 400 \, \mu m\); arranged in modules of \(6.24 \, cm \times 2.14 \, cm\) with 46080 pixels, read out by 16 chips, each serving an array of 18 by 160 pixels. Total 80 million pixels, area 1.7 m\(^2\).

The modules are overlapped on the support structure to give hermetic coverage. The thickness of each layer is about 2.5% of a radiation length at normal incidence. Typically three pixel layers are crossed by each track. The readout chips will suffer \(\geq 300 \, kGy\) of ionizing radiation and \(\geq 5\times10^{14}\) neutrons per cm\(^2\) over ten years.
A Pixel Module

Cross section:

Diagram showing guard geometry near edges of module, designed to operate safely with bias voltages of beyond 700V.
The ATLAS SemiConductor Tracker

The SCT system is designed to provide eight precision measurements per track in the intermediate radial range, contributing to the measurement of momentum, impact parameter and vertex position.

In the barrel SCT eight layers of silicon microstrip detectors provide precision point in the $r$-$\phi$ and $z$ coordinates, using small angle stereo to obtain the $z$-measurement. Each silicon detector is $6.36 \times 6.40$ cm$^2$ with 780 readout strips of 80 µm pitch. The barrel modules are mounted on carbon-fiber cylinders at radii of 30.0, 37.3, 44.7, and 52.0 cm.

The end-cap modules are very similar in construction but use tapered strips with one set aligned radially. The SCT covers a pseudorapidity-range $< 2.5$. 

Momentum Resolution of Tracking Systems

Tracking detectors serve to measure and identify the primary vertex (origin) of the charged tracks, and to determine the track momenta. To find the momentum resolution, consider a typical collider tracker consisting of $N$ concentric tracking barrel detectors, centered on the beam pipe, giving $N$ equidistant measurement points ("hits") with measurement errors $\sigma$ (transverse to the track’s direction).

Assume the existence of a solenoidal uniform magnetic field $B$ directed along the beam direction. The tracks will have a length $L_{xy}$ measured between first and last hit in the transverse (to the beams) plane. The momentum resolution from the measurement uncertainty in this simplest case can be shown to be:

$$\text{Resolution} = \frac{1}{N} \frac{1}{\sigma} \sqrt{L_{xy}}$$

This equation is valid for $N \geq 8$. A more general expression for arbitrary spacing and hit uncertainties is given in the reference below, which also discusses multiple scattering limits when the detectors have significant density.

Transition Radiation Tracker

Ginzburg and Frank (1944): Transition radiation is emitted at dielectric interfaces;

- Radiated energy is proportional to $\gamma = E/M$.
- Small number of photons: $\sim \alpha$ per interface; photon energy: 5-20 keV; forward emission.
- Use Xe instead of Ar in gaseous (drift) detectors (straws) for efficient photon detection (by photo-ionization) $\Rightarrow$ electron ID

- The radiators are formed by the straw walls and C-fiber mats

![Graph showing drift velocity vs. electric field]

![Schematic view of straw manufacturing process]

The TRT straw made from multilayer Kapton films and reinforced with carbon fibre bundles.
TRT: Electron vs. Pion Discrimination

Barrel part: 52,544 straws, 144 cm long, 4 mm Ø, oriented parallel to the beam.
Two end-caps: each 122,880 straws, 37 cm long, radial to the beam axis.

Particles cross 35–40 straws in $|\eta| \leq 2$, providing continuous tracking at larger radii.
Hit resolution: 140 µm
TRT: Electron vs. Pion Discrimination

Differential energy spectra in a single straw with radiator for test beam data for 20 GeV electrons and pions. The points are from the ATLAS simulation (left)

Pion efficiency vs. Electron efficiency at 20 GeV (right)
ATLAS Calorimeters


**Calorimetry**

Calorimeters are sensitive to all particles that have the electromagnetic and/or strong interaction. They operate by partially converting the energy $E$ of the incident particle into a large collection of secondary, tertiary, etc. particles in a “showering” process. This showering is the result of successive electromagnetic and/or strong interactions of the particles created.

Between interactions, the *charged* particles lose energy by ionization, as discussed in the chapter above.

The showering stops when the energy of every particle produced dips below the threshold at which it can no longer produce secondaries; e.g. for an electron this is when its energy is below the bremsstrahlung threshold ($\sim$10 MeV), for an photon when its energy is below the pair production threshold, for a proton when its energy is not enough to produce an additional pion in a strong interactions, and so on ...

Below-threshold charged particles lose their remaining energy by ionization. All unstable particles produced in the showering eventually decay and stable particles get stopped and absorbed.

The shower development is a purely statistical process where, for a given type of calorimeter, the number of particles produced $N$ is proportional to the incident energy $E$ and nothing else. The signal derived from the calorimeter is proportional to $N$, and therefore the resolution of a calorimeter goes as:

$$\frac{\sigma(E)}{E} \approx \frac{\sqrt{N}}{N} \approx \frac{1}{\sqrt{N}}$$
Calorimetry

With segmentation in “depth” the characteristic evolution of the shower can be measured. In addition to improving the measurement of energy, this is very useful method to distinguish electron showers from photon showers (which start later), from hadron showers (which are much “coarser”), and from muons (which typically just leave a minimum ionization trail). This is exemplified in Figure 40.

With transverse segmentation, the spatial position of the shower and its direction can be measured, which complements the tracking measurement by giving an “energy-vector”.

The fast response time of a typical calorimeter (10 ns – 1 μs) is very useful for triggering purposes: at the LHC with a total pp inelastic cross section of $\sigma_{\text{inel}} \approx 80 \text{ mb} = 0.8 \times 10^{-25} \text{ cm}^2$ and at a luminosity of $10^{34} \text{ cm}^{-2}\text{s}^{-1}$, the raw rate of interactions is 800 MHz, i.e. about 20 interactions per bunch crossing (crossing rate is 40 MHz).

A fast and efficient trigger is required to capture the few interactions of interest. This is done in stages, where the first and fastest $O(\mu s)$ stage selects energy clusters above a certain threshold, or muons passing through the calorimeter into the muon system, etc., followed by more sophisticated selections $O(\text{ms})$ based on tracking, calorimeter cluster profiles, and precise muon momentum measurement, etc. A final event filter is used with the full reconstruction software to select the event sample going to disk (100 Hz).
Electromagnetic and Hadronic Calorimetry

Because the electromagnetic showers from electrons and photons are much more compact than hadron-induced showers (generally $X_0$ is much smaller than the hadronic interaction length $\lambda$), the calorimeter is divided into two main depth layers, one optimized for the measurement electromagnetic showers, the “electromagnetic” (EM) calorimeter, followed by a coarser “hadronic” calorimeter.

We discuss these two types in the following sections.

Figure 40. Slice of the ATLAS detector perpendicular to the beam pipe (bottom). For illustration, the typical signatures of several particle types in the detector is shown.

Note, hadrons may equally well interact in the EM calorimeter!
Electromagnetic Calorimetry

For electrons and photons with energies above 10 MeV, the processes of pair production and bremsstrahlung together create a dense electromagnetic cascade of electrons, positrons and photons, which evolves until the individual particles in the shower drop below an energy of about 10 MeV at which point they lose energy mostly by ionization and the photoelectric and Compton processes.

This is a statistical process: after a distance of \( \lambda = X_0 \ln 2 \), on average half the initial energy of an electron is lost to a bremsstrahlung photon, i.e. we find two “particles” sharing the original energy.

After a second step \( \lambda \), the bremsstrahlung photon makes an electron-positron pair and the electron radiates another bremsstrahlung photon, making a total of four particles with a quarter of the energy each.

In the third step, the number of particles doubles again each having half the energy, and so forth.
EM Shower Development

At a depth of $lX_0$, we have $N(l) = 2^{l/\ln2} = e^l$ electrons, positrons, and photons with average energy $E(l) = E/e^l$ each. The showering process continues until a particle's energy $E(l)$ falls below the threshold energy $\varepsilon$ for pair creation and bremsstrahlung; from then onwards only the photoelectric and ionization losses occur.

This is a simplification: we neglected ionization loss, assumed pair and bremsstrahlung cross sections to be constant, and the pair-production length to equal $X_0$!

From this we easily derive the position of the shower maximum:

$$E(l_{\text{max}}) = E/e^{l_{\text{max}}} = \varepsilon,$$

and $l_{\text{max}} = \ln(E/\varepsilon)$ and $N(l_{\text{max}}) = E/\varepsilon$.

The total charged particle track length (of electrons and positrons) in the calorimeter is:

$$L[X_0] = \frac{2}{3} \sum_{l=1}^{l_{\text{max}}} N(l) = \frac{2}{3} \sum_{0}^{l_{\text{max}}} e^l = \frac{2}{3} \frac{e^{l_{\text{max}}+1}}{e-1} = \frac{2}{3} \frac{e E/\varepsilon}{e-1} = \frac{2}{3} \frac{1}{1-e^{-1}/\varepsilon} \frac{E}{\varepsilon} = 1.05 \frac{E}{\varepsilon} \propto E$$

Figure 41. A simulated electron shower in the ATLAS Barrel accordion Pb-LAr sampling calorimeter.
**EM Shower Development**

Clearly, the total track length $L$ is composed of minimum ionizing electrons and positrons, two-thirds of all particles in the shower.

At 10 MeV, electrons and positrons are minimum ionizing and the ionization signal in the calorimeter is therefore strictly proportional to $L$!

In the idealized case, the shower reaches a maximum and then abruptly stops; in practice, the shower shows an exponential rise in number of particles, rises to a broad maximum, and has a slower exponential decline.

In the Table below, *fitted shower properties* for electrons and photons of energy $E$ are listed, together with the results of our simplified model:

<table>
<thead>
<tr>
<th>property</th>
<th>Electron</th>
<th>Photon</th>
<th>Simple Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shower maximum $l_{max}$ [$X_0$]</td>
<td>$1.0 \left[ \ln(\frac{E}{\epsilon}) - 0.5 \right]$</td>
<td>$1.0 \left[ \ln(\frac{E}{\epsilon}) + 0.5 \right]$</td>
<td>$1.0 \ln(\frac{E}{\epsilon})$</td>
</tr>
<tr>
<td>Center of Gravity $[X_0]$</td>
<td>$l_{max} + 1.4$</td>
<td>$l_{max} + 1.7$</td>
<td></td>
</tr>
<tr>
<td>Number of $e^+$ and $e^-$ at max</td>
<td>$0.3 \frac{E}{\epsilon} \left[ \ln(\frac{E}{\epsilon}) - 0.37 \right]^{-\frac{1}{2}}$</td>
<td>$0.3 \frac{E}{\epsilon} \left[ \ln(\frac{E}{\epsilon}) - 0.31 \right]^{-\frac{1}{2}}$</td>
<td>$(2/3) \frac{E}{\epsilon}$</td>
</tr>
<tr>
<td>Total track length $L$ [$X_0$]</td>
<td>$\frac{E}{\epsilon}$</td>
<td>$\frac{E}{\epsilon}$</td>
<td>$1.05 \frac{E}{\epsilon}$</td>
</tr>
</tbody>
</table>
Electron Shower in Iron

Powerful software programs have been developed to simulate EM and Hadronic showers in exquisite detail.

Figure 42: An EGS4 simulation of a 30 GeV electron-induced cascade in iron. The histogram shows fractional energy deposition per radiation length (the curve is a gamma-function fit to the simulated distribution.) Circles indicate the number of electrons with total energy greater than 1.5 MeV crossing planes at $X_0/2$ intervals (scale on right), and the squares the number of photons with $E \geq 1.5$ MeV crossing the planes (scaled down to have same area as the electron distribution).

Figure 42. From: “Passage of particles through matter,” (and references therein) by H. Bichsel, D.E. Groom, and S.R. Klein, Particle Data Group (C. Amsler et al., Phys. Lett. B 667, Ch 27 (2008)).
Longitudinal and Transverse Shower Profile

Because the electromagnetic interaction is very well understood, simulations by GEANT4 or EGS, of an electromagnetic shower closely resemble experimental data and are widely used to predict EM calorimetric response ...

The critical energy for Fe is about 22 MeV. In our simple model, the shower maximum occurs at ln(30 GeV/22 MeV) = 7.2 \(X_0\). From Fig. 42 it is seen that, in order to fully contain a 30 GeV EM shower, the calorimeter must have a depth of at least 20 \(X_0\). The containment depth grows logarithmically with incident energy. One sees that more photons are created than electrons, contrasting with our simple model of one bremsstrahlung event per \(X_0\), and that photons persist to deeper depth, indicating that electrons (and positrons) keep radiating below the critical energy.

The transverse profile of the EM shower is approximately Gaussian in the shower core, and 90% of the shower’s energy is contained inside a cylinder with radius equal to the “Molière radius” \(R_M\):

\[
R_M = X_0 \frac{m_e \sqrt{4\pi/\alpha}}{E_c} \approx X_0 \frac{21.2 \text{ MeV}}{E_c}
\]

e.g. \(R_M(\text{Fe}) \approx X_0(\text{Fe}) = 1.76 \text{ cm}\).

In summary: the EM shower is smooth on a scale characterized by the radiation length \(X_0\). The radiation length varies approximately like \(A/Z^2\).
EM Resolution

We argued that the measured energy in a (sufficiently deep) calorimeter is proportional to the incident energy and is determined in a statistical process of measuring secondary particles in the shower. The measurement of the shower energy, i.e. the energy of the secondaries, can be done in a variety of ways:

• Measurement of the ionization loss of the secondaries.
• Measurement of the Cerenkov light generated by the secondaries in a transparent medium.
• Measurement of the scintillation light generated by the secondaries.
EM Resolution

The measurement may be done in a homogenous calorimeter (NaI, Leadglass, BGO, ...) or in a “sampling” calorimeter in which thin slabs of absorber and detector material alternate.

In a sampling calorimeter, the absorber is a high-Z material (Pb, U, ...) divided in slabs 0.5 – 1.0 $X_0$ thick; the thin sampling detector consist of scintillator or an ionization detector (Liquid Ar, ...).

The resolution is therefore a statistical process characterized by the measurement of some finite energy unit $\Delta E$, which may be the typical ionization loss by a particle crossing a Liquid Argon gap in a sampling calorimeter, or the energy required to produce a photo-electron in a uniform NaI crystal. Typically $\Delta E < E_c = \varepsilon$ (the cut-off energy).

Uniform calorimeters have thus a resolution limit set by this energy “unit”:

$$\frac{\sigma(E)}{E}_{\text{sampling}} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{E/\Delta E}} = \sqrt{\frac{\Delta E}{E}} = 3.2\% \sqrt{\frac{\Delta E [\text{MeV}]}{E [\text{GeV}]}}$$

For instance, Leadglass (glass loaded by 50–60% PbO), operates by the measurement of Cerenkov light emitted by the electrons and positrons with $\beta \geq 1/n$ in the shower. Typically, in a 1 GeV EM shower a 1000 photoelectrons are created in a photo-multiplier photocathode viewing a leadglass block. The threshold energy in Leadglass is about $\Delta E \approx 0.7$ MeV, and thus the number of track segments is $1.0 \text{ GeV/0.7 MeV} = 1.4 \times 10^3$ and $\sigma_{\text{sampling}} \approx 2.7\%$. The photoelectron statistics in an additional 3.2%, to be added in quadrature: $\sigma_{\text{total}} \approx 4.1\%$ at 1 GeV.
EM Resolution

In all cases, many effects contribute to a finite energy resolution. In most cases a single source dominates: in sampling calorimeters this is typically the fluctuation in the number of gap crossings (“samplings”) \( N = \frac{L}{L_{\text{cell}}}(\text{total charged track length})/L_{\text{cell}}(\text{cell length}) \), because of the statistical nature of the bremsstrahlung and pair production processes. This effect dominates by far over the ionization loss statistics.

Several effects worsen the resolution further:

• Shower leakage: if the EM shower is not fully contained, energy escapes the calorimeter (longitudinally or sideways), leading to a degradation of resolution.

• The secondaries cross the cells under angles determined by multiple scattering, increasing the per-cell path length and decreasing the statistics of gap crossings and worsening the resolution.

• We have ignored issues of electronic noise, calorimeter non-uniformity of response, variations in absorber and gap thickness, etc.
Example Calculation for a Pb-LAr Sampling Calorimeter

Consider an EM calorimeter consisting of 50 sheets of Pb, each with 3.00 mm thickness. The Pb sheets are separated by "gaps" of 6.00 mm filled with Liquid Argon. Electrons of 50.0 GeV are directed into the calorimeter perpendicular to the sheets.

The following data are used: for LAr \((Z=18)\): \(\rho_{\text{LAr}} = 1.396 \text{ g/cm}^3\), \(X_0(\text{Ar}) = 19.55 \text{ g/cm}^2 = 14.00 \text{ cm}\), and \(dE/dx_{\text{min}}(\text{LAr}) = 2.1 \text{ MeV/cm}\). For Pb \((Z=82)\): \(\rho_{\text{Pb}} = 11.35 \text{ g/cm}^3\), \(X_0(\text{Pb}) = 6.37 \text{ g/cm}^2 = 0.56 \text{ cm}\), and \(dE/dx_{\text{min}}(\text{Pb}) = 12.8 \text{ MeV/cm}\). The calorimeter is at least 15 cm/0.56 cm = 27 \(X_0\) deep, enough to contain the shower fully.

Consider the calorimeter a mixture of Pb and LAr based on relative weight fractions. The mixture has a density of \(\rho = 4.71 \text{ g/cm}^3\) and a \(X_0 = 7.33 \text{ g/cm}^2 = 1.56 \text{ cm}\): 
\[
1/X_0(\text{mix})[\text{cm}] = 3 \text{ mm/9 mm} \times 
1/X_0(\text{Pb})[\text{cm}] + 6 \text{ mm/9 mm} \times 1/X_0(\text{LAr})[\text{cm}].
\]

The cut-off energy \(\epsilon = 610 \text{ MeV}/(Z+1.24)\); for Pb \((Z=82)\): \(\epsilon(\text{Z=82}) = 7.3 \text{ MeV}\) and 31.7 MeV for Ar; and thus the Lead determines the cut-off energy!

The \(dE/dx\) per “cell” (1 sheet Pb + 1 gap LAr) equals 5.10 MeV, i.e. \(\Delta E = 8.84 \text{ MeV/}X_0\).

The average energy of the shower particles at a depth \(l[X_0]\) is now: \(E_l = (E+2/3\Delta E)/e^l - 2/3\Delta E\).

The depth where the shower particles reach \(\epsilon\) is modified to \(l_{\text{max}} = \ln[(E+2/3\Delta E)/(\epsilon+2/3\Delta E)]\).

The total track length becomes: \(L = 1.05(E+2/3\Delta E)/(\epsilon+2/3\Delta E) = 3.98 \times 10^3 X_0 = 6.21 \times 10^3 \text{ cm}\).

The number of cell crossings \(N\) is thus \(N=L[\text{cm}]/(0.900 \text{ cm}) = 6.90 \times 10^3 \) crossings, which must equal the number of LAr gap crossings. The energy resolution will be dominated by fluctuations in this number: \((N)^{-2/3} = 1.2\%\) for an \(E=50 \text{ GeV}\) electron, i.e. a sampling term of 8.5%/\(\sqrt{E}\).
ATLAS EM Calorimeter

In order to obtain spatial information about the energy deposits in the calorimeter, i.e. energy vectors, a calorimeter is segmented in both longitudinal and transverse directions, see Fig. 44.

Test beam results of an ATLAS Barrel LAr calorimeter module are shown in Fig. 43 at pseudorapidity $\eta=0.40$.

Figure 43. Longitudinal (3 depths) and transverse (as indicated by bins in $\Delta\eta$ and $\Delta\phi$) segmentation of an ATLAS Barrel accordion calorimeter module.

Figure 44. Longitudinal (3 depths) and transverse (as indicated by bins in $\Delta\eta$ and $\Delta\phi$) segmentation of an ATLAS Barrel accordion calorimeter module.
EM-Barrel Signal Electrodes and PS-Barrel

Barrel Electrodes Spaced by honeycomb mats

<table>
<thead>
<tr>
<th>Compartment</th>
<th>$\Delta \eta$</th>
<th>$\Delta \phi$</th>
<th>$X_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Back</td>
<td>.050</td>
<td>$2\pi/256$</td>
<td>1.4–7</td>
</tr>
<tr>
<td>Electrodes</td>
<td>.025</td>
<td>$2\pi/256$</td>
<td>16.5–19</td>
</tr>
<tr>
<td>Middle</td>
<td>.025</td>
<td>$2\pi/256$</td>
<td>16.5–19</td>
</tr>
<tr>
<td>Presampler</td>
<td>.200/8</td>
<td>$2\pi/64$</td>
<td>.08–.15</td>
</tr>
</tbody>
</table>

PSB samples in 11 mm LAr the energy lost upstream (~1.7 $X_0$)

$\eta = 0.8$

$\eta = 1.475$
EM Barrel Calorimeter

Half-Barrel

Absorber

Internal ring

Cryostat rail

Cryostat cold wall

External ring

Cooling loop

Presampler sector (in its housing)

Presampler modules
Performances

Table 8. Various Calorimeter technologies, the experiment in which used, their depth, resolution performance, and the year this was achieved.

<table>
<thead>
<tr>
<th>Technology (Exp)</th>
<th>Depth [X₀]</th>
<th>Energy resolution</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaI(Tl) (Crystal Ball)</td>
<td>20</td>
<td>2.7%/\sqrt{E}</td>
<td>1983</td>
</tr>
<tr>
<td>Bi₄Ge₃O₁₂ (BGO) (L3)</td>
<td>22</td>
<td>2%/\sqrt{E} ⊕ 0.7%</td>
<td>1993</td>
</tr>
<tr>
<td>CsI (KTeV)</td>
<td>27</td>
<td>2%/\sqrt{E} ⊕ 0.45%</td>
<td>1996</td>
</tr>
<tr>
<td>CsI(Tl) (BaBar)</td>
<td>16–18</td>
<td>2.3%/\sqrt{E} ⊕ 1.4%</td>
<td>1999</td>
</tr>
<tr>
<td>CsI(Tl) (BELLE)</td>
<td>16</td>
<td>1.7% for Eₓ &gt; 3.5 GeV</td>
<td>1998</td>
</tr>
<tr>
<td>PbWO₄ (PWO) (CMS)</td>
<td>25</td>
<td>3%/\sqrt{E} ⊕ 0.5% ⊕ 0.2%/E</td>
<td>1997</td>
</tr>
<tr>
<td>Lead glass (OPAL)</td>
<td>20.5</td>
<td>5%/\sqrt{E}</td>
<td>1990</td>
</tr>
<tr>
<td>Liquid Kr (NA48)</td>
<td>27</td>
<td>3.2%/\sqrt{E} ⊕ 0.42% ⊕ 0.09%/E</td>
<td>1998</td>
</tr>
<tr>
<td>Scintillator/depleted U (ZEUS)</td>
<td>20–30</td>
<td>18%/\sqrt{E}</td>
<td>1988</td>
</tr>
<tr>
<td>Scintillator/Pb (CDF)</td>
<td>18</td>
<td>13.5%/\sqrt{E}</td>
<td>1998</td>
</tr>
<tr>
<td>Scintillator fiber/Pb (KLOE)</td>
<td>15</td>
<td>5.7%/\sqrt{E} ⊕ 0.6%</td>
<td>1995</td>
</tr>
<tr>
<td>Liquid Ar/Pb (NA31)</td>
<td>27</td>
<td>7.5%/\sqrt{E} ⊕ 0.5% ⊕ 0.1%/E</td>
<td>1988</td>
</tr>
<tr>
<td>Liquid Ar/Pb (SLD)</td>
<td>21</td>
<td>8%/\sqrt{E}</td>
<td>1993</td>
</tr>
<tr>
<td>Liquid Ar/Pb (H1)</td>
<td>20–30</td>
<td>12%/\sqrt{E} ⊕ 1%</td>
<td>1998</td>
</tr>
<tr>
<td>Liquid Ar/U (DØ)</td>
<td>20.5</td>
<td>16%/\sqrt{E} ⊕ 0.3% ⊕ 0.3%/E</td>
<td>1993</td>
</tr>
<tr>
<td>Liquid Ar/Pb accordion (ATLAS)</td>
<td>25</td>
<td>10%/\sqrt{E} ⊕ 0.4% ⊕ 0.3%/E</td>
<td>1996</td>
</tr>
</tbody>
</table>
Hadronic Calorimeters

Hadrons interact via the EM and the strong interactions. Because hadronic masses are much larger than the electron mass, \( m_\pi = 139.6 \text{ MeV} \approx 270 \ m_e \), bremsstrahlung does not occur until ultra-high energies, and only ionization and nuclear interactions play a role.

The strong interaction cross section is very complicated at low energy, but becomes approximately constant (per target nucleon) at high energy (> 10 GeV). The characteristic length, the “interaction length” \( \lambda_i \) is much larger than \( X_0 \):

\[
\lambda_i \approx 35 \ \text{gcm}^{-2} \ A^{1/3}
\]

This can be understood as follows. From Sect.2.2 we have \( \lambda_i^{-1} = (1/A)\rho N_A \sigma \), where \( \sigma \approx 50 \ \text{mb} = 50 \times 10^{-27} \ \text{cm}^2/\text{nucleon} \). Because of shielding, there are effectively \( A^{2/3} \) nucleons per nucleus of atomic number \( A \), i.e. \( \sigma = 50 \times 10^{-27} \ \text{cm}^2 \ A^{2/3} \). Combining, we find:

\[
\rho \lambda_i = \frac{A}{N_A \sigma} \approx \frac{A \ \text{g/mol}}{N_A \ 50 \times 10^{-27} \ \text{cm}^2 \ A^{2/3}} = \frac{A^{1/3} \ \text{g/mol}}{6.022 \times 10^{23} \ \text{mol}^{-1} \ 50 \times 10^{-27} \ \text{cm}^2 \ A^{2/3}} = 33 A^{1/3} \ \text{g/cm}^2
\]

Thus the interaction length depends on \( A^{1/3} \), very different from the dependence of \( X_0 \).
Hadronic Interactions in Matter

Hadronic interactions at “low” energies are theoretically incalculable because they are too complex: “bags” of quarks and gluons interact.

Simplification emerges only at high energy, when typically a single “hard” scattering dominates: quasi-free quark-quark, quark-gluon, and gluon-gluon interactions are calculable in the framework of Quantum Chromo-Dynamics (QCD).

Often new hadrons are produced; a few examples:

\[ \pi^- p \rightarrow \pi^0 n; \quad \pi^0 \rightarrow \gamma \gamma \]
\[ \rightarrow \rho^- p; \quad \rho^- \rightarrow \pi^0 \pi^- \]
\[ \rightarrow \pi^- \Delta^+; \quad \Delta^+ \rightarrow n \pi^+ \]
\[ \rightarrow \pi^- p \pi^+ \pi^- \]
\[ \rightarrow \cdots \]

In the calorimetric measurement of hadrons, strong interactions must therefore be simulated with the help of extensive energy dependent tables.

Total cross sections for \( \pi^\pm p, K^\pm p, \) and \( p^\pm p \) vs. \( s \) (center-of-mass energy squared).

Hadronic Showers

The showering differs vastly from the EM shower for electrons and photons in that the strong interaction gives rise to a very large variety of final states: multi-pions (typically dominating the final state), Kaons, protons/neutrons, nuclear fragments, photons, etc..

Whenever a $\pi^0$ is produced (on average in equal numbers to charged pions), the $\pi^0$ decays immediately into two photons. Therefore, any hadronic shower contains a large amount of EM showering as well.

The size of the EM component versus the hadronic component has very large fluctuations because the ratio depends strongly on the result of the very first interactions.

The calorimeter response to slow nuclear fragments and neutrons depends strongly on the absorber material; neutrons are effectively stopped by light nuclei, but bounce around elastically in heavy materials. In the heaviest materials (Uranium), neutrons may cause fission, creating additional fragments and neutrons.

Because of this the response of a calorimeter to a hadron ($\pi^\pm, p, ...$) of a given energy is not the same as its response to an electron of the same energy!
ATLAS Hadronic Calorimeter

Consequently, the resolution for hadrons is much worse than for EM radiation, with a statistical term around $80\%/\sqrt{E}$.

In sampling calorimeters one may choose the absorber and detector materials separately, and thus one has one more parameter to possibly “compensate” for the differences in response to hadronic and EM cascades.

Figure 46. Resolution of the Liquid Argon EM + Hadronic forward calorimeters for pions in the ATLAS 2002 beam test. The curve is a fit of the form $82\%/\sqrt{E}$. The G4 points are GEANT4 simulations.
ATLAS Hadronic Calorimeter: TileCal

TileCal Module being “wired”:
Top of the wedge is pointing to the beams. Scintillating tiles (insert) are positioned in the slots, with wavelength-shifting readout fibers routed to photomultiplier tubes.
ATLAS Muon Spectrometer

With a combination of mechanical accuracy of each chamber and external position monitoring (laser alignment), the “Monitored” Drift Tube (MDT) system achieves a sagitta accuracy of 60 µm, i.e. a momentum resolution of 10% at $p_T = 1$ TeV/c. The 1,150 MDT chambers are made from 354,000 tubes and cover an area of 5,500 m².

MDT: Al tube with 3.0 cm Ø, sense wire 50 µm Ø at 3kV, with Ar-CO₂ (93%-7%) at $p=3$ bar: $v_D \approx 21$ µm/ns.
ATLAS Barrel Toroid: Installation of 8 SuperConducting Coils

11/04/2004: Installation of the first coil

08/26/2005: Installation of the last coil

$I=20.5 \, \text{kA}, \int B dl \approx 3 \, \text{Tm}$
The ATLAS Detector – The Design
ATLAS Detector in the Cavern

The reality ...

100 m underground, in Summer 2008
Cosmic Ray Data Taking

Many months of cosmic ray muons to debug detector ...
9/10/2008: First single-beam event seen in the ATLAS detector! (interaction by Beam 1 in the upstream collimator)