Problem XI.1

a. Show by working out the calculation in more detail, that the $L_{int}$ (Lecture Notes #12, slide 26) works out as indicated for the couplings of the photon, $W^\pm$ bosons, and the $Z$ boson to the first generation leptons. Eliminate the coupling $g'$ by the use of the Weinberg angle $\theta_W$ and coupling $g$.

b. Write $L_{int}$ for the $(u,d)_L SU(2)_L$ doublet and the $u_R$ and $d_R$ singlets, and work out their couplings to the photon, $W^\pm$ bosons, and the $Z$ boson. Eliminate the coupling $g'$ by the use of the Weinberg angle $\theta_W$ and coupling $g$.

c. Argue that the Fermi constant must be $g^2\sqrt{2}/8M^2$, with $M$ the mass of the $W$-boson.

Hints: no hints.

Solution: no solution yet

Problem XI.2

Prove the following relations:

i. $\Gamma(W^+ \rightarrow ud)/\Gamma(W^+ \rightarrow e^+ \nu_e) = 3 \cos^2 \theta_{\text{Cabibbo}}$

ii. $\Gamma(W^+ \rightarrow us)/\Gamma(W^+ \rightarrow e^+ \nu_e) = 3 \sin^2 \theta_{\text{Cabibbo}}$

Hints: no hints

Solution: no solution yet

Problem XI.3

i. Given the values of the Fermi constant $G_F=1.16637(1)\times10^{-5} \text{ GeV}^{-2}$, the electric charge $e=1.60217653(14)\times10^{-19} \text{ C}$, and the $Z$-boson mass $M_Z=91.1876(21) \text{ GeV/c}^2$, calculate the values of the $U(1)_Y$ and $SU(2)_L$ couplings $g$ and $g'$, the $W$-boson mass $M_W$, the Weinberg angle $\sin^2 \theta_W$, and the vacuum expectation value of the Higgs field $v$.

ii. Compare your calculated values of $M_W$ and the Weinberg angle $\sin^2 \theta_W$ with their experimentally determined values.

Hints: See notes; watch out for the units, e.g. for $e$!

Solution: no solution yet