Problems:
VII.1 Calculate the center-of-mass cross section \( \frac{d\gamma}{d\delta^*} \) for the process of spinless \( e^- e^+ \rightarrow e^- e^+ \) scattering, where \( \delta^* \) is the center-of-mass scattering angle between the outgoing \( e^- \) and the incoming \( e^- \). How does it differ from the spinless \( e^- \mu^+ \rightarrow e^- \mu^+ \) scattering you calculated before?

Hints:
rework the expressions \( s^*u/\ell \) etc. in terms of \( \delta^* \)

Solution:
There are two contributing diagrams; a "\( t \)-channel" exchange diagram (as before) plus an annihilation diagram or "\( s \)-channel" diagram. Both amplitudes must be added together (note: we use \( m_1=m_2=m_3=m_4 \) in the last line):

\[
-i M_{\beta} = -i \frac{-e^2(p_1+p_3)_\mu(-p_2-p_4)\mu}{(p_1-p_3)^2} + \frac{-e^2(p_1-p_2)_\mu(-p_4+p_3)\mu}{(p_1+p_2)^2} \]
\[
= -i(-e^2) \left[ -\frac{s-u}{t} - \frac{t-u}{s} \right] \quad (I.1)
\]

Using:
\( s = 4E^{*2}, \quad t = -2E^{*2}(1 - \cos \theta^*), \quad u = -2E^{*2}(1 + \cos \theta^*) \)

one finds:

\[
|M_{\beta}|^2 = e^4 \left( \frac{3 + \cos \theta^*}{1 - \cos \theta^*} \right) = e^4 \left( \frac{3 + \cos^2 \theta^*}{1 - \cos \theta^*} \right)^2
\]

For the differential cross section:

\[
\frac{d\sigma}{d\Omega^*}_{\text{CMS, all masses ignored}} = \frac{1}{2\pi} \frac{d\sigma}{d\theta^*} = \frac{1}{64\pi^2 s} |M_{\beta}|^2 = \frac{e^4}{64\pi^2 s} \left( \frac{3 + \cos^2 \theta^*}{1 - \cos \theta^*} \right)^2
\]

Thus, this differs from the previous \( e^- \mu^+ \rightarrow e^- \mu^+ \) scattering that was calculated before by the \( \cos^2 \theta^* \) term instead of the \( \cos \theta^* \) we had there before…