Honors Classical Physics I

PHY141
Lecture 35
Volume & Temperature
Heat (Flow)
The Ideal Gas Law
H6B Up and down-stream
Heat Flow

Heat flow $dQ/dt$ [W] can occur by three different means, often occurring simultaneously:

**Conduction** is flow of energy through materials (e.g. transported by quasi-free electrons in metals, and by lattice vibration waves (phonons):

$$\frac{dQ}{dt} = k A \frac{(T_H - T_L)}{L},$$

- with $A$ the cross sectional area of the heat conductor (perpendicular to the heat flow),
- and $L$ the total length of the heat conductor (parallel to the heat flow),
- and $T_H$ ($T_L$) the high and low temperatures at the two sides of the material,
- and the proportionality constant $k$ is the material-dependent HEAT CONDUCTIVITY of the material; SI units: [W/K/m]
- Also used: the HEAT RESISTANCE $R \equiv 1/k$ (“R”-value, USA units [ft$^2$ °F hr/BTU])

**Convection** is the primary mechanism of heat transport in fluids and gases. Approximately: $dQ/dt \propto A (T_H - T_L)^{5/4}$
Heat Flow

**Radiation** is energy flow by emission or absorption of electromagnetic radiation (light, infra-red radiation, ...):

\[ H = \frac{dQ}{dt} = eA\sigma T^4, \]

- with \( A \) the total area of the body emitting or absorbing the radiation,
- and \( T \) the Kelvin temperature of the emitting body,
- the proportionality constant is \( \sigma = 5.6705 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4} \), the universal Stefan-Boltzmann constant,
- and \( e \) is the material-dependent emissivity or absorption coefficient \((0<e<1)\). For rough and black materials \( e \approx 1 \)

Note that **EVERY** object at \( T_K > 0 \) emits radiation; including the surroundings; the **NET** heat flow into a body is thus:

\[ H = \frac{dQ}{dt} = eA\sigma(T_s^4 - T_b^4), \]

- with \( T_b \) is the Kelvin temperature of the body, and \( T_s \) the Kelvin temperature of the surroundings!
- Typically, radiation loss is small compared to other losses ...
the heat flow through an outside wall (size Length×Height) decreases to half the value ...

A. if I half the inside temperature ...
B. if the wall doubles in Area (Length×Height) ...
C. if the wall thickness doubles ...

A. 33%  B. 33%  C. 33%
Example

One end of a 20 cm long uniform copper bar is kept at 20.00 K, while the other end is blackened and exposed to surroundings at 500 K. Assuming only radiative flow occurs between the surroundings and the blackened end of the bar, calculate the temperature of the blackened end. (Hint: the temperature will be close to 20 K because copper is a good heat conductor)

- The net radiative heat flow into the blackened end (emissivity $e=1$) of the bar equals the heat conduction through the bar:

$$H_{\text{rad}} = H_{\text{cond}}$$

$$\sigma A (500^4 - T^4) = kA(T - 20.00)/L$$

$$5.6705 \times 10^{-8} (500^4 - T^4) = (1670/0.20) \Delta T$$

for $T \approx 20$ K, $T^4 << 500^4$ and can be ignored on the left-hand side!

Thus:

$$\Delta T = 5.6705 \times 10^{-8} (500^4) / (1670/0.20) = 0.42$$

$\Rightarrow T = 20.42$ K
Equation of State

First: remember mass and density:
- mass: \( m = n \cdot M \) [kg], with \( n \) the number of moles [mol] and \( M \) the molar mass [kg/mol]
- density: \( \rho \equiv \frac{m}{V} \) [kg/m\(^3\)]
- 1 mol of a stuff is defined as \( N_A = 6.022 \times 10^{23} \) molecules of it (or as the number of atoms in exactly 12 grams of Carbon-12)

Equation of State for an “IDEAL GAS”:
\[ pV = nRT_K \]
- where \( p \) is pressure, \( V \) is volume, \( T \) is the Kelvin temperature, \( n \) is # moles, and \( R = 8.3145 \text{ J/mol/K} \) is the “GAS CONSTANT”
- for 1 mol of an (ideal) gas at STP (Standard Temperature \( T = 0 \text{ °C} \), and Pressure \( p = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} \)):
  \[ V = 1 \times 8.3145 \times 273.15 / 1.013 \times 10^5 = 22.41 \text{ L} \]
- \( \Rightarrow pV/T \) is constant for fixed amount of gas!

for not-so-ideal gas (van der Waals):
\[ (p - a(n/V)^2)V(1 - b(n/V)) = nRT_K \]
- e.g. \( \text{CO}_2 \): \( a = 0.364 \text{ Nm}^4/\text{mol}^2, \ b = 4.27 \times 10^{-5} \text{ m}^3/\text{mol} \)
- Average distance between molecules at STP:
  \[ \frac{V}{N_A} = 22.4 \times 10^{-3} / 6.022 \times 10^{23} = 3.7 \times 10^{-26} \text{ m}^3 = L^3 \Rightarrow L = 3.3 \text{ nm ( ~30-60× atomic dimension) } \]
State and Phase Diagrams

Substances exist in three phases:
- solid,
- liquid,
- gas/vapor

Changing volume $V$, pressure $p$, and or temperature $T$ allows us to “scan” the whole phase space, which is a surface in 3-DIMENSIONAL space!
Example

Given the density of outside air (at STP) $\rho_{\text{cold}}$, calculate the required temperature of the hot air $T_{\text{hot}}$ in a hot-air balloon of volume $V$, and mass $m_{B}$ (balloon skin, burners, payload) for take-off...

- to take off, the weight of hot air, payload, etc must be cancelled by the Buoyant force from the surrounding cold air ...
- Equilibrium ($m_{\text{hot}}$ is the mass of the hot air):

$$m_{\text{hot}}g + m_{B}g = \rho_{\text{cold}}Vg$$

$$\Rightarrow m_{\text{hot}} = \rho_{\text{cold}}V - m_{B} \Rightarrow m_{\text{hot}}/V = \rho_{\text{hot}} = \rho_{\text{cold}} - m_{B}/V$$

- treating the air as an ideal gas, the number of moles $n$ of hot air in the balloon as function of temperature is:

$$pV = nRT = (m/M)RT$$

$$\Rightarrow T_{\text{cold}} = p(V/m)(M/R) = (pM/R)/\rho;$$

$$\Rightarrow T_{\text{hot}}/T_{\text{cold}} = \rho_{\text{cold}}/\rho_{\text{hot}}; \quad T_{\text{hot}} = T_{\text{cold}}/(1 - m_{B}/\rho_{\text{cold}}V$$
The unit of the product $pV$ is …

A. N
B. Nm³
C. J
D. N/m

A. 25%  B. 25%  C. 25%  D. 25%
Kinetic Gas Theory

Derivation of $pV = nRT$: Assume a gas of $nN_A$ molecules, all of mass $m$. Assume these to be point-like, and colliding elastically with the walls of a box with $V = L^3$, and with each other...

- the molecules bounce off the wall (wall’s mass $>> m$) and thereby exert a minute force...
- Let’s calculate: consider a molecule $i$ with velocity $v_i$; then the force on the molecule by the wall is:

$$F_{xi} = \frac{\Delta p_{xi}}{\Delta t} = m \frac{\Delta v_{xi}}{\Delta t} = m \left( \frac{-v_{xi}}{\Delta t} - v_{xi} \right) = -\frac{2mv_{xi}}{\Delta t}$$

- the reaction force on the wall is thus $2mv_{xi}/\Delta t$.
- We find the TIME-AVERAGED force on the wall when we take $\Delta t$ as the time between successive collisions with the (same) wall:

$$\Delta t = 2L_x/v_{xi} \Rightarrow F_{xi} = \frac{mv_{xi}^2}{L_x}$$

- there are $nN_A$ molecules in the volume; for each molecule: $v_i^2 = v_{ix}^2 + v_{iy}^2 + v_{iz}^2$, but the averages of the squared velocity components of the molecules are equal:

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} \equiv \overline{3v_x^2}$$

- the total force by $nN_A$ molecules is:

$$F_x = nN_A \frac{mv_x^2}{L} = nN_A \frac{mv^2}{3L} \Rightarrow p = \frac{F_x}{L^2} = nN_A \frac{mv^2}{3L^3} \Rightarrow pV = nN_A \frac{mv^2}{3} = n \frac{2}{3} N_A K$$