Honors Classical Physics I

PHY141
Lecture 17
Momentum and Collisions
Rotational Kinematics - Recap

Please set your Clicker Channel to 21
Center-of-Mass

• In a collision, *when no external forces act*, the Center-of-Mass momentum does not change, and the Center-of-Mass position keeps constant velocity...

• For a system (e.g. the body of a diver), the motion of the Center-of-Mass follows from the net force:

\[ F_{\text{net}} = \sum_j F_j = Ma_{CM} \]

• We’ll discuss the relative motion of the parts (e.g. rotations) around the CM a bit later ....
Calculate the CM of the disk-with hole
(Hint: use symmetry)

A. CM is at \( x=0, \ y=0 \)

B. CM is at \( x=-R/6, \ y=0 \)

C. CM is at \( x=-R/4, \ y=0 \)

D. CM is at \( x=-R/3, \ y=0 \)

E. CM is at \( x=-R/2, \ y=0 \)

\[
x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i}; \quad y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i}
\]
CM of the disk-with hole

\[ x_{cm} = \frac{\sum \limits_{i} m_i x_i}{\sum \limits_{i} m_i}; \quad y_{cm} = \frac{\sum \limits_{i} m_i y_i}{\sum \limits_{i} m_i} \]

\[ x_{CM} = \frac{m \times 0 + \left(-\frac{m}{4}\right) \times \frac{R}{2}}{m + \left(-\frac{m}{4}\right)} = -\frac{R}{6}, \quad y_{CM} = 0 \]
In rocket motion, we have another example of a collision:

\[ \sum \vec{F}_{ext} = M \vec{a}_{CM} \]

- where the system of rocket + expelled gases has a constant total mass \( M \)
- and \( \vec{F}_{ext} \) is, for instance, gravity, ...

Rocket equation:

\[
\vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{\vec{P}_f - \vec{P}_i}{dt} = \frac{m(\vec{v} + d\vec{v}) + \vec{u} \, dm - (m+dm) \vec{v}}{dt} \\
= \frac{md\vec{v} + \vec{u} \, dm - \vec{v} \, dm}{dt} = m \frac{d\vec{v}}{dt} + (\vec{u} - \vec{v}) \frac{dm}{dt} \\
\Rightarrow m \frac{d\vec{v}}{dt} = ma_{\text{rocket}} = F_{\text{rocket}} = F_{ext} - (\vec{u} - \vec{v}) \frac{dm}{dt} = F_{ext} - v_{\text{rel}} \frac{dm}{dt} = F_{ext} + F_{\text{thrust}}
\]

Before burn of \( dm \) in fuel:

\[ m+dm \]

\[ \vec{v} \]

After burn of \( dm \) in fuel:

\[ m \]

\[ \vec{u} \]

\[ \vec{v} + d\vec{v} \]

**Note vectors:**

\[ \vec{u}_{\text{Gas, Sun}} = \vec{v}_{\text{Gas, Rocket}} + \vec{v}_{\text{Rocket, Sun}} \]
Does it make a difference to the rocket what happens to the ejected gases after they have left the rocket motor?

A. No, not at all …
B. No, as long as the gases hit the Earth’s atmosphere, the propulsion remains unchanged …
C. Yes, if the gases cannot effectively push against the ground, the rocket will accelerate less …

In the previous slide we did NOT concern ourselves with the gases after ejection! $F_{\text{rocket}}$ does not depend on it!
A fully fueled rocket (21×10^3 kg, including 15×10^3 kg fuel), spews out fuel at \( dm/dt = 190 \text{ kg/s} \) with exhaust speed of \( v_{rel} = 2800 \text{ m/s} \). It is fired upwards near sea level; assume constant gravity.

• **calculate speed vs. \( t \), and \( v_{final} \):**

\[
m \frac{dv}{dt} = F_{ext} - (u - v) \frac{dm}{dt} = F_{ext} - v_{rel} \frac{dm}{dt} \quad \Rightarrow \quad dv = \frac{F_{ext}}{m} dt - v_{rel} \frac{dm}{m}
\]

\( \text{Note: } v_{rel} \text{ is directed down!} \)

\[
\Rightarrow \int dv = - \int g \, dt + v_{rel} \int \frac{dm}{m} \quad \Rightarrow \quad v(t) = \left( -gt - v_{rel} \ln \frac{m(t)}{m_0} \right) \hat{j}
\]

• **burn time is:** \( t_{burn} = 15\times10^3 \text{ kg} / 190 \text{ kg/s} = 79 \text{ s} \).

• **Final speed is thus:** 

\[
v_f = -9.80 \times 79 \text{ m/s} - 2800 \text{ m/s} \ln \frac{6,000 \text{ kg}}{21,000 \text{ kg}}
\]

\[
= +2,700 \hat{j} \text{ m/s}
\]

• **Note: CM of rocket-plus-gases is “falling down” with } a = g ! **

- while the rocket by itself is accelerating up !!
Rotational Kinematics - Recap
Kinematics of Rotation

• We revisit in more detail rotational kinematics
  - Motion: \( \mathbf{v}(t) = \mathbf{v}_0 + \dot{\mathbf{v}}/t, \quad \mathbf{s}(t) = \mathbf{s}_0 + \mathbf{v}_0 t + \frac{1}{2} \dot{\mathbf{v}}/t^2 \)
    - true for any (curved) path with constant acceleration \( \dot{\mathbf{v}} \) along (//) the path!
  - Circular motion with constant angular acceleration:
    divide the motion formulae by \( R \) to get:
    \[ \omega(t) = \frac{d\theta}{dt} = \omega_0 + \alpha t, \quad \theta(t) = \frac{s(t)}{R} = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2, \quad \alpha = \frac{d\omega}{dt} = \frac{d^2 \theta}{dt^2} \]
    • speed and acceleration of the “rim”: \( v// = \omega R, \quad a// = a_{\text{tan}} = \alpha R \)
    • centripetal acceleration (\( \perp \) to path): \( a\perp = a_{\text{rad}} = v^2/R = \omega^2 R \)
    • \( \Rightarrow \) total acceleration: \( \mathbf{a} = a// \hat{i} + a\perp \hat{j} \) (2 components!)
    • Uniform Circular Motion: \( a// = 0 \)

• Rotations are ONLY of interest for EXTENDED objects; idealized point-like objects have effectively no rotation...
• Extended objects are either RIGID or more or less Non-Rigid (i.e. floppy)
  in this course we’ll discuss mostly RIGID objects and their rotation...

\( \Rightarrow \) Of paramount importance: the (location of the) AXIS OF ROTATION
an object rotates with 60 rpm (revolutions per minute); its angular velocity is …

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60\times2\pi \text{ rad/min} = 2\pi \text{ rad/s} = 6.283 \text{ rad/s}
Example

A baseball pitcher spins on his axis before he throws his ball; at a certain instant he spins at 8.0 rad/s, with an angular acceleration of 48 rad/s², while the ball goes through a curve of radius $R = 60$ cm (this sets the axis of rotation!)

- **Q:** calculate the acceleration $a$ of the ball at that instant.

\[
\omega = 8.0 \text{ s}^{-1}, \quad \alpha = 48 \text{ s}^{-2}
\]

\[
a_{\perp} = a_{\text{rad}} = a_c = \frac{v_{||}^2}{R} = \omega^2 R, \quad a_{||} = a_{\text{tan}} = \alpha R
\]

\[
a = a_{\text{rad}} \mathbf{i} + a_{\text{tan}} \mathbf{j}; \quad \text{e.g.:} \quad a = |a| = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2}
\]
Rotational Kinematics:
A bicycle wheel, radius $R = 0.35$ m, has rolled in $t = 4.0$ s without slipping a distance on the road of $s = 42$ m with constant linear (and thus also constant angular) acceleration. The angular velocity at $t=4.0$ s is $\omega(t) = 3.18$ rev/s.

- Q: what was the initial angular velocity?

$$\omega(t) = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{\omega - \omega_0}{t}$$

$$\Delta \theta = \frac{s}{R} = \omega_0 t + \frac{1}{2} \alpha t^2 \quad \Rightarrow \quad \frac{s}{R} = \omega_0 t + \frac{1}{2} (\omega - \omega_0) t = \frac{\omega + \omega_0}{2} t$$

$$\omega_0 = \frac{2s}{Rt} - \omega = 60 - 3.18 \times 2\pi = 40 \text{ rad/s}$$

- Q: what is the angular acceleration?

- i.e. it is an angular deceleration!

- Q: calculate the linear (de-)acceleration:

$$a = \alpha R = -1.75 \text{ m/s}^2$$