**Vectors:**

\[ \mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} \]

Scalar Product ("Dot" Product)

\[ \mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z = AB \cos \theta_{A,B} = AB \mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z \]

Vector Product ("Cross" Product)

\[ \mathbf{A} \times \mathbf{B} = (A_x B_y - A_y B_x) \mathbf{i} + (A_x B_z - A_z B_x) \mathbf{j} + (A_y B_z - A_z B_y) \mathbf{k} \]

direction: "right-hand rule"

Kinematics:

- velocity \( v \): acceleration \( a \):
  \[ v = ds/dt \quad (\text{speed } = \mathbf{v}) \quad a = dv/dt \]

- Linear motion with constant \( a \):
  \[ v = v_0 + at \quad s = s_0 + v_0 t + \frac{1}{2} at^2 \]
  eliminating \( t \):
  \[ v^2 = v_0^2 + 2a(s - s_0) \]

- rotation angle \( \theta \): rotation radius \( R \):
  \[ \theta = s(=\text{arc length}) \quad R = B(t) \]
  \[ \theta = \theta_0 + \omega t + \frac{1}{2} \alpha t^2 \]
  \[ \omega = \omega_0 + \alpha t \]

- Angular velocity \( \omega \): angular acceleration \( \alpha \):
  \[ \omega = \omega_0 + \alpha t \]
  \[ \omega = \theta_0 + \alpha t + \frac{1}{2} \alpha t^2 \]

- Centripetal acceleration \( a_c \):
  \[ a_c = \frac{v^2}{R} \]

- Center-of-Mass Position of System (mass \( M \))
  \[ \mathbf{r}_{cm} = \sum m_i \mathbf{r}_i / \sum m_i = \sum m_i \mathbf{r}_i / M \]

- Moment of Inertia \( I \):
  \[ I = \sum m_i r_i^2 \quad r_i = \text{distance between rotation axis and } m_i \text{(dm)} \]

- Parallel-Axis Theorem
  \[ I = I_{cm} + Md^2 \]

- Perpendicular-Axis Theorem for a Planar (flat) body in the x-y plane:
  \[ I = I_z + I_x + I_y \]

- Moment \( p \):
  \[ p = \sum p_i = \sum m_i v_i \]

- Angular Momentum \( L \):
  \[ L = \sum \mathbf{r}_i \times m_i v_i = \mathbf{r}_i \times m_i v_i \]

- Kinetic Energy \( K \):
  \[ K = \sum \frac{1}{2} m_i v_i^2 \]

- Moving axis:
  \[ K = \frac{1}{2} \sum m_i v_i^2 + \frac{1}{2} I_{cm} \omega_{cm}^2 \]

- Fixed axis:
  \[ K_{tot} = \frac{1}{2} I_{cm} \omega_{cm}^2 \]
**Forces** \([N=kgm/s^2]\) and consequences:

<table>
<thead>
<tr>
<th>(F_i = dp_i/dt = ma_i)</th>
<th>(F_{tangent} = -F_{normal})</th>
<th>(\Sigma F = dL/dt = \Gamma)</th>
</tr>
</thead>
</table>

**Force of Gravity** between \(M\) and \(m\), at center-to-center distance \(r\):

\[ F_g = GMm/r^2 \quad \text{(downwards, } g = 9.80 \text{ m/s}^2) \]

**Force of a Spring** (spring constant \(k\)):

\[ F_s = -kx \quad \text{(opposes compression/stretch \(x\))} \]

**Friction**: static: \(F_f \leq \mu_N\), kinetic: \(F_f = \mu_N N\), opposes motion, \(\mu\)-friec coefficient, \(N\)-normal force

**Torque**:

\[ \tau = \text{cross product of } \mathbf{R} \times \mathbf{F} \quad \text{(right-hand rule)} \]

**Equilibrium** & **Collisions**:

If \(\Sigma F = 0\), then \(\Delta p_{net} = 0\); if \(\Sigma F = 0\), then \(\Delta L_{net} = 0\)

**Impulse by a force** \(F\) over a time interval:

\[ J_t = \int F dt \]

**Work done by a force** \(F\) over a trajectory:

\[ W_t = \int F \cdot dx \]

**Work done by a torque** \(\tau\) over a rotation angle \(\theta\):

\[ W_t = \int \tau \cdot d\theta \]

**Work-Kinetic Energy relationship** (from \(\Sigma F=m\alpha\)):

\[ W_{kin} = \Sigma W_i = \Delta K = K_f - K_i \]

**Power** \(P\) \(= W/t\):

\[ P_t = dW/dt = F \cdot v = \tau \cdot \omega \]

**Potential Energy** \(U\) of a **conservative force** \(F\):

\[ U_r = -W_F ; \quad e.g. \; U_k = -GMm/r, \; U_k = v^2/2k \]

**Work by Non-Conservative forces** - **Total Energy**:

\[ W_{nc} = \Delta E = E_t - E_i = K_f + U_f - (K_i + U_i) \]

---

**Periodic Motion** (period \(T\), frequency \(f\), angular freq. \(\omega\)) for restoring forces, e.g. spring force \(F = -kx\) and mass \(m\):

- **pendulum** (distance \(L\) between cm and rotation point):
  \[ x(t) = A \cos(\omega t + \phi), \; \omega = 2\pi/T, \; f = 1/T \]
  \[ \omega = \sqrt{k/m}, \; \phi = \arcsin \left( \frac{mg}{kL} \right) \]

**Damping** force \(F = b \cdot \dot{x}\) and damping:

\[ A = F_{max} \cdot \sqrt{(k/m)^2 + (\omega b)^2} \]

**Traveling Waves** \((T, f, \omega, \text{wave length and number } k, \text{propagation velocity } v = \lambda/T = sf\), e.g. waves in a **string** of linear mass \(m/L\) under tension \(F\):

- **sound waves** in air \((T = 20^\circ C, \rho = 1\) atm) (intensity \(I\)):
  \[ x(t) = A \cos(kx - \omega t + \phi), \; \lambda = 2\pi/T, \; k = \pi\lambda, \; v = \sqrt{\mu/\rho \; \text{sound level } dB = 10 \log(10)} \]

**Doppler effect** (velocity of source \(v_s\) listener \(v_l\) signs):

\[ v_s = v_l \quad \text{(against)} \]

**superposition principle**: displacements add \(\rightarrow\) interference.

- **Standing Waves** - interference of wave with itself

**Fluids** (Buoyant force \(= \rho V g\) of displaced fluid)

- **continuity equation** (incompressible):
  \[ \rho V = \text{constant throughout fluid} \]

- **Bernoulli's equation** (incompressible, frictionless):
  \[ p = \rho g y + \frac{1}{2} \rho v^2 = \text{constant throughout fluid} \]
Temperature scales (Kelvin K, Celsius °C, °F) \( T = T_K = 273.15 + T_C; \ T_C = \frac{5}{9} T_F + 32 \ °F \)

Expansion of solids and liquids: \( L_2 = L_0(1 + \alpha \Delta T), \ V_2 = V_0(1 + \beta \Delta T), \ \beta = 3a ; \ \Delta T = T - T_0 \)

Flow rate \( \dot{Q} = \frac{dQ}{dt} \)
- Conduction: \( \dot{Q} = -k A \frac{dT}{dx} \)
- Radiation: \( \dot{Q} = \sigma A T^4 \)

Ideal gas law (number of moles \( n \), \( T \) in kelvin)

- \( pV = nRT - nN_kkT; \ N_k = 6.02 \times 10^{23} \ \text{mol} \)
- \( R = 8.31 \ \text{J/K/mol}; \ k = R/N_k = 1.38 \times 10^{-23} \ \text{J/K} \)

Kinetic-molecular model (\( K_n \) the average translational kinetic energy of the molecules, \( \bar{v}_m \) the root-mean-square speed): \( K_n = \frac{1}{2} m \bar{v}_m^2 = \frac{3}{2} kT \)

Heat (\( Q \)) added to fluid/solid (specific heat \( c \), latent heat \( L \)) in same phase, and for a phase change f. v.:
- \( Q_{f \rightarrow s} = mc(T_f - T_s) \) (no phase change)
- \( Q_{f \rightarrow s} = mL \) (change to fluid or vapor)

Heat (\( Q \)) added to a gas at constant \( p \) or \( V \):
- \( C_p, \gamma \) molar heat capacity:
- \( C_V = \frac{1}{2} R \) for ideal monatomic gas;
- \( C_V = \frac{3}{2} R \) for ideal di-atomic gas

1st Law: \( Q = \Delta E_{\text{int}} + W \) (heat \( Q \) added to the gas; change in internal energy \( \Delta E_{\text{int}} \), work \( W \) done by the gas [J]): \( \Delta E_{\text{int}} = nC_p \Delta T; \ W = \int pdV \)

Thermodynamic Processes:
- Isochoric: \( V \) constant; isobaric: \( p \) constant;
- Isothermal: \( T \) constant \( \rightarrow pV \) constant; adiabatic \( Q = 0 \) \( \rightarrow pV \) constant

Reversible process: process that remains in quasi-equilibrium during the change, and can therefore be fully reversed

State variable: variable determined uniquely by the state of the system, \( n, p, V, T, U, S \)

Entropy \( dS = dQ/T \) irreversible 2nd Law: \( \Delta S = \int dQ/T \) irreversible \( \geq 0 \)

Cycle in a \( pV \)-diagram (efficiency \( \eta \)):
- \( \eta = \frac{W_{\text{net}}}{Q_{\text{in}}} \leq \eta_{\text{Carnot}} \)

Carnot cycle (between two isotherms \( (T_H, T_C) \) and two adiabats):
- \( \eta_{\text{Carnot}} = 1 + Q_H/Q_H = 1 - T_C/T_H \)

Problems …
Two identical loudspeakers separated by distance $d$ emit 200 Hz sound waves along the $x$-axis. As you walk along this axis, away from the speakers, you don’t hear anything even though both speakers are on.

- What are three possible values for $d$? Assume a sound speed of 340 m/s.

\[ d = n \lambda + \frac{\lambda}{2} \]
\[ \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{200 \text{ Hz}} = 1.715 \text{ m} \]

\[ n = 1, 2, 3, \ldots \]
Lake Erie is prone to remarkable seiches - standing waves that slosh water back and forth in the lake basin from the west end at Toledo to the east end at Buffalo.

The figure shows smoothed data for the displacement from normal water levels along the lake at the high point of one particular seiche. 3 hours later the water was at normal levels throughout the basin; 6 hours later the water was high in Toledo and low in Buffalo.

- What is the wavelength of this standing wave?

- What is the frequency?

- What is the wave speed?
A 40-cm-long tube has a 40-cm-long insert that can be pulled out. A vibrating tuning fork is held next to the tube.

As the insert is slowly pulled out, the sound from the tuning fork creates standing waves in the tube when the total length is 43.5 cm, 58.0 cm, and 72.5 cm.

- What is the frequency of the tuning fork?
  The speed of sound in the air is 343 m/s.
A typical LASIK laser emits a 1.0-mm-diameter laser beam with a wavelength of 193 nm. Each laser pulse lasts 15 ns and contains 1.5 mJ of light energy.

- What is the power of one laser pulse?

\[
P = \frac{\Delta E}{\Delta t} = \frac{1.5 \text{ mJ}}{15 \text{ ns}} = \frac{15 \times 10^{-3} \text{ J}}{15 \times 10^{-9} \text{ s}} = 1 \times 10^{5} \text{ W}
\]

- During the very brief time of the pulse, what is the intensity of the light wave?

\[
I = \frac{P}{\pi R^2} = \frac{1 \times 10^{5} \text{ W}}{\pi (1 \times 10^{-3} \text{ m})^2} = \frac{9}{\pi} \times 10^{6} \text{ W/m}^2
\]
What is the sound intensity level of a sound with an intensity of $2.8 \times 10^{-6}$ W/m²?

\[ P = 10 \log_{10} \frac{I}{I_0} = 10 \text{ dB} \log_{10} \frac{2.8 \times 10^{-6}}{I_0} \]
A whistle you use to call your hunting dog has a frequency of 21 kHz, but your dog is ignoring it. You suspect the whistle may not be working, but you can’t hear sounds above 20 kHz. To test it, you ask a friend to blow the whistle, then you hop on your bicycle.

- In which direction and what speed should you ride (toward or away from your friend) to know if the whistle is working?

\[ f_L = f_S \frac{v + v_L}{v - v_S} \]

\[ 21 \text{ kHz} = 21 \text{ kHz} \left( 1 + \frac{v_L}{v} \right) \]

\[ \sim 1 \text{ kHz} = \frac{v_L}{v} \Rightarrow v_L = \frac{v}{21} \quad \text{or} \quad v = -16.5 \text{ m/s} \]
A point on a string undergoes simple harmonic motion as a sinusoidal wave

- When a sinusoidal wave with speed 28 m/s, wavelength 30 cm and amplitude of 2.0 cm passes, what is the maximum speed of a point on the string?

\[ y = A \cos \left( \frac{2\pi}{\lambda} x - \omega t \right) = A \cos(\omega t) \]

\[ \frac{dy}{dt} = -\omega A \sin(\omega t) \quad \Rightarrow \quad v_{\text{max}} = \omega A \]

\[ \omega = \omega_{\text{max}} = 2\pi f = \frac{2\pi}{T} = \frac{2\pi v}{\lambda} \]
The amplitude of an oscillator decreases to 65.8% of its initial value in 30.0 s.

- What is the value of the time constant?

\[ y = A_0 e^{-\frac{t}{\tau}} \sin(\omega t) \]

\[ A(\tau = 30.5) = 0.66 = e^{-\frac{30.5}{\tau}} \]

\[ \ln 0.66 = -\frac{30.5}{\tau} \Rightarrow \tau = \frac{30.5}{\ln 0.66} \]
A 230 g air-track glider is attached to a spring. The glider is pushed in 10.2 cm against the spring, then released. A student with a stopwatch finds that 14 oscillations take 12.5 s.

- What is the spring constant?
The New England Merchants Bank Building in Boston is 152 m high. On windy days it sways with a frequency of 0.14 Hz, and the acceleration of the top of the building can reach 2.0% of the free-fall acceleration, enough to cause discomfort for occupants.

- What is the total distance, side to side, that the top of the building moves during such an oscillation?
The blood pressure at your heart is approximately 100 mm of Hg. As blood is pumped from the left ventricle of your heart, it flows through the aorta, a single large blood vessel with a diameter of about 2.5 cm. The speed of blood flow in the aorta is about 60 cm/s. Any change in pressure as blood flows in the aorta is due to the change in height: the vessel is large enough that viscous drag is not a major factor.

- What is the maximum height the brain could be above the heart?
- The density of blood is 1060 kg/m³.

\[
\begin{align*}
\text{The blood pressure at your heart is approximately } & 100 \text{ mm of Hg. As blood is pumped from the left ventricle of your heart, it flows through the aorta, a single large blood vessel} \\
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& \text{is large enough that viscous drag is not a major factor.} \\
& \text{What is the maximum height the brain could be above the heart?} \\
& \text{The density of blood is 1060 kg/m}^3. \\
\end{align*}
\]
The blood pressure at your heart is approximately 100 mm of Hg. The aorta has a diameter of about 2.5 cm and the speed of blood flow is about 60 cm/s. As the blood moves through the circulatory system, it flows into successively smaller and smaller blood vessels.

Blood flows in the capillaries at the much lower speed of approximately 0.7 mm/s. The diameter of capillaries and other small blood vessels is so small that viscous drag is a major factor.

- Estimate the total cross sectional area of the capillaries

\[
\text{Continuity Equation: } \quad \frac{A_r \cdot v_a}{v_c} = \frac{A_{\text{cap }} \cdot v_c}{v_c} \\
\Rightarrow A_{\text{cap}} = A_{\text{aorta}} \cdot \frac{v_a}{v_c} = \pi R_{\text{aorta}}^2 \cdot \frac{60 \text{ cm/s}}{0.7 \text{ cm/s}} = \ldots
\]
Suppose that in response to some stimulus a small blood vessel narrows to 90% of its original diameter. If there is no change in the pressure across the vessel, what is the ratio of the new volume flow rate to the original flow rate?

For narrow pipes, turbulent: viscosity becomes important.

\[ \text{Poisson's equation } \Rightarrow \text{volume flow rate } \propto R^4 \text{ if } \Delta p \text{ is fixed} \]

\[ D' = 0.9 D \Rightarrow \text{volume flow rate new} = \frac{\pi d x (0.9)^4}{0.66} = 0.66 \]
Air flows through the tube shown in the figure. Assume that air is an ideal fluid.

- What is the air speed at point 1?

\[
p + \rho g y + \frac{1}{2} \rho v^2 = \text{constant}
\]

\[
p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \quad (y_1 = y_2)
\]

From figure: \[p_x = p_1 + 10 \text{ cmHg} = p_1 + \rho Hg \text{Hg}
\]

\[
\frac{1}{2} \rho v_1^2 = \rho Hg \text{Hg} + \frac{1}{2} \rho v_2^2
\]

\[
\frac{1}{2} \rho v_1^2 = \rho Hg \text{Hg} + \frac{1}{2} \rho \left( \frac{A_1}{A_2} \right) v_2^2
\]

\[
\rho = \ldots
\]
A child’s water pistol shoots water through a 1.0 mm-diameter hole. If the pistol is fired horizontally 65 cm above the ground, a squirt hits the ground 1.2 m away.

- What is the volume flow rate during the squirt? Ignore air resistance.

Volume flow rate is \( \dot{V} \) (dimension \( m^3/s \)).

We know the area of the hole: \( A = \pi R^2 \) with \( R = 0.5 \text{ mm} \).

What is \( v \)? Must be sufficient enough to reach 1.2 m.

In the \( y \)-direction: \( v_y = 0 \), \( a_y = -g \) \( \Rightarrow y - y_0 = \frac{v_y^2}{2a_y} \Rightarrow -0.65 = -1 \) \( \Rightarrow e = \quad \).

In the \( x \)-direction: \( v_x = v_x e \) \( \Rightarrow v_x = \quad \Rightarrow \quad \).
A wooden block of 1.0 m x 1.0 m x 0.10 m is floating in water. The density of the wood is 800 kg/m³, and for water 1000 kg/m³.

- How much of the wooden block is above the water as it floats?

- What maximum mass can be added on top of the block before it sinks?

\[
\frac{V_{\text{under water}}}{V_{\text{total}}} = \frac{\rho_{\text{water}}}{\rho_{\text{wood}}} = 80\% \implies 20\% \text{ is above the water}
\]

\[
m = V_{\text{block}} (\rho_{\text{water}} - \rho_{\text{wood}}) = 0.10 \text{ m}^3 \times 2000 \text{ kg/m}^3 - 800 \text{ kg/m}^3 = 20 \text{ kg}
\]
A monatomic gas follows the process shown in the figure.

- How much heat is needed for process 1→2 and for process 2→3?

\[ Q_{1→2} = nC_v \Delta T = \frac{nC_v}{R} (P_2V_2 - P_1V_1) \]

\[ T_2 = \frac{P_2V_2}{nR} \]

\[ Q_{2→3} = nC_v \Delta T = \frac{nC_v}{R} (P_3V_3 - P_2V_2) \]

\[ V_3 = V_2 = 9 \times 10^{-5} \text{ m}^3 \]
A rectangular trough, 1.9 m long, 0.70 m wide, and 0.50 m deep, is completely full of water. One end of the trough has a small drain plug right at the bottom edge.

- When you pull the plug, at what speed does water emerge from the hole?

\[
p + pg_y + \frac{1}{2} \rho u^2 = \text{constant}
\]

**Boundary conditions:**
- **Trough:** \( y = 0, \ p = p_e = 1 \, \text{atm} \)
  \[
p_e + g + \frac{1}{2} \rho u_e^2 = \]
- **Hole:** \( p = p_e \) (outside the hole), \( y = -0.5 \, \text{m} \)
  \[
p_e - p_e H + \frac{1}{2} \rho u_e^2 = \]

**Continuity:** \( A_1v_1 = A_2v_2 \) \( \Rightarrow v_1 = \frac{A_2}{A_1} v_2 = \text{small fraction} \) \( v_2 \ll \frac{1}{g} \times 0.5 \, \text{m} \)

\( \Rightarrow v_1^2 \) can be ignored compared to \( v_2^2 \)!

\( \Rightarrow v_e^2 \) can be ignored compared to \( v_2^2 \)!

\[
\frac{1}{2} \rho v_e^2 = 0 = -pg_H + \frac{1}{2} \rho v_e^2 \quad \Rightarrow \quad v_e = \sqrt{g H} = \ldots
\]
25 g of liquid water is placed in a flexible bag, the air is excluded, and the bag is sealed.

It is then placed in a microwave oven where the water is boiled to make steam at 100 °C.

What is the volume of the bag after all the water has boiled? Assume that the pressure inside the bag is equal to atmospheric pressure ...

\[ \text{We have } 25 \text{ g of } H_2O \rightarrow 25 \text{ g of steam at } T=373 K \]

\[ pV = nRT \]

\[ R = 8.31 \text{ J/mol K}, \quad T = 373 K, \quad P = 1 \text{ atm} = 101.3 kPa \]

\[ n = \frac{m}{M_{\text{mol}}} \]

\[ n = \frac{25g}{18g/mol} \approx 1.39 \text{ mol} \]

\[ V = \frac{nRT}{P} = \frac{1.39 \text{ mol} \times 8.31 \text{ J/mol K} \times 373 K}{101.3 \times n^3 P_a} \approx 0.042 \text{ m}^3 = 42 \text{ L} \]

\[ \leq 10 \text{ Gal} \]
A bird in 15 m/s horizontal flight uses 10 W provided by fat stored in his body. 1 g fat is equivalent to 9.4 Cal. Assume that 30% actually goes to the mechanical work of flying.

- How far will this bird fly on 4 g of fat?
- First: how much mechanical energy coming from 4 g of fat:
  
  \[ E_{\text{mech}} = 4 \text{ g} \times \frac{9.4 \text{ Cal} \times 4190 \text{ J/Cal}}{1 \text{ g}} \times 30\% = 4.73 \times 10^4 \text{ J} \]

- At a 10 W = 10 J/s rate of work (power), how long will this last?
  
  \[ P = \frac{W}{\Delta t} \Rightarrow \Delta t = \frac{E_{\text{mech}}}{P} = \frac{4.73 \times 10^4 \text{ J}}{10 \text{ J/s}} = 4.73 \times 10^3 \text{ s} \]

- The distance the bird travels in this time:
  
  \[ D = v\Delta t \Rightarrow D = 15 \text{ m/s} \times 4.73 \times 10^3 \text{ s} = 71 \text{ km} \]

A girl of mass \( m = 60 \text{ kg} \) springs from a trampoline with an initial upward velocity of \( v_0 = 8.0 \text{ m/s} \). At height \( h = 2.0 \text{ m} \) above the trampoline, the girl grabs a box of mass \( M = 15 \text{ kg} \).

- What is the speed \( v \) of the girl immediately before she grabs the box?
  
  \[ mgh + \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 \Rightarrow v = \sqrt{v_0^2 - 2gh} = 4.98 \text{ m/s} \]

- What is the speed of the girl immediately after she grabs the box?
  
  \[ mv = (m + M)V \Rightarrow V = \frac{vm}{(m + M)} = 3.98 \text{ m/s} \]

- Is this "collision" elastic or inelastic?
  
  box and girl stick together \( \Rightarrow \) completely inelastic!

- What is the maximum height (as measured with respect to the top of the trampoline) that the girl with box reaches?
  
  \[ \frac{1}{2}(m+M)V^2 + (m+M)gh = (m+M)gh' \Rightarrow h' = h + V^2/(2g) = 2.81 \text{ m} \]
A \( M=1500 \text{ kg} \) car is going through a horizontal circular curve of \( R=150 \text{ m} \) radius, at a speed \( v=30 \text{ m/s} \).

- Calculate the magnitude and direction of the net force on the car.

Circular motion at constant speed

\[
a_c = \frac{v^2}{R}
\]

\[
F_c = ma_c = m \frac{v^2}{R}
\] towards the center
A satellite in circular orbit rotates around the Earth exactly \( N = 3.0 \) times each day.
- Calculate its height above the surface.
a mass $m$ is hanging off a string of length $L=1.5$ m, and is going through a horizontal circle. The string makes a $\theta=30^\circ$ angle with the vertical.

- Calculate the speed $v$ of $m$ in the circle.
A $M=3.0 \text{ kg}$ uniform disk with radius $R=20 \text{ cm}$, is spinning at 300 rpm.

- What force of friction applied to the rim must be applied to stop the disk in $\Delta t=3.0 \text{ s}$?

\[ \omega_f = 300 \text{ rpm} = 300 \frac{\text{rev}}{60} = 10 \pi \text{ rad/s} \]

\[ \omega_f(t = 3.0\Delta t) = \omega = \omega_f + \alpha \Delta t \Rightarrow \alpha = \frac{\omega - \omega_f}{\Delta t} = \frac{10 \pi}{3.0} \text{ rad/s}^2 \]

\[ \text{Torque due to } F_k : \quad R F_k = I \alpha \quad \text{and} \quad I = \frac{1}{2} M R^2 \]

\[ F_k = I \alpha / R = \frac{1}{2} M R \left( \frac{10 \pi}{3.0} \text{ rad/s}^2 \right) = \ldots. \]
A wheel is touching a step with height equal to half the wheel’s radius.

- What minimum force $F$ is required to lift the wheel up the step?

4 forces: $\mathbf{m}g$, $\mathbf{F}$, and forces $n$ and $N$.

1. To lift the wheel up: $n = 0$ (lift off from floor).
2. I’m not interested in $N \Rightarrow$ choose rotation point on the step’s corner $\mathbf{r}$.
3. When just lifting: $n = 0$ and equilibrium ...

\[
\sum \tau_i = 0 = R \mathbf{mg} \sin(\theta) - \frac{R}{2} \mathbf{F}
\]

$\Rightarrow F = 2 \mathbf{mg} \sin(2\theta) = \ldots$
Lightweight ropes are wrapped around a composite pulley constructed out of two connected cylinders of masses $m_1=4.5 \text{ kg}$ and $R_1=0.10 \text{ m}$, and $m_2=18 \text{ kg}$ and $R_2=0.20 \text{ m}$.

Weights are hung as indicated in the figure.

- What is the acceleration and in which direction?
A uniform barrel of mass \( M = 600 \text{ kg} \) and radius \( R = 0.60 \text{ m} \) is held in place on a slope by a polypropylene rope attached to the rim. The coefficient of static friction is \( \mu_s = 0.25 \).

a) What is the tension in the rope?

b) What is the steepest slope for which this is possible?

c) What is the smallest rope diameter that can be used? (PE: \( F_{\text{max}}/A = 20 \times 10^6 \text{ N/m}^2 \))
Suppose the woman in the figure is 50 kg, and the board she is standing on has a 10 kg mass.

- What is the reading on each of the scales?

- Net force and net torque are zero!

\[ N_1 + N_2 - (m + M)g = 0 \Rightarrow N_1 + N_2 = (m + M)g \text{ makes sense!} \]

(2) Choose pivot at \( N_1 \):

\[-(m)Mg - (1.5M)mg + N_2 (2m) = 0 \Rightarrow N_2 = \frac{(m + 1.5M)g}{2.0} \]

Find \( N_1 \) either by using \( N_2 \) in equation 1, or by choosing the pivot at \( N_2 \).
How close to the edge of the $m=50 \text{ kg}$ table, see the figure, can a $M=60 \text{ kg}$ person stand before it tips over?

\[ D = 2.00 \text{ m} \]

\[ d = 0.50 \text{ m} \]

\[ x \]

\[ \Sigma F = 0 \text{ and } \Sigma T = 0 \]

Just not tipping over $\Rightarrow N = 0$ ?

We are not interested in $N \Rightarrow$ choose pivot at $N$ ?

\[ \Sigma T = 0 : \]

\[ (1M - 0.50 \text{ m}) mg - (0.50 \text{ m} - x) Mg = 0 \]

\[ \frac{1}{2} mg(\frac{1}{2} - x) Mg \Rightarrow \frac{1}{2} - x = \frac{m}{2M} \Rightarrow x = \frac{1}{2} - \frac{m}{2M} = 0.3 \text{ m} \]
A woman weighing 600 N does a pushup from her knees, as shown in the figure.

1. What are the normal forces of the floor on each of her hands?
2. What are the normal forces of the floor on each of her knees?

\[ \Sigma F_z = 0 \]

Choose pivot at knees:

\[ - (0.8 \text{ m}) 2N_1 + (0.5 \text{ m}) W = 0 \Rightarrow 1.6 N_1 = 0.5 W \Rightarrow N_1 = \ldots \]

Find \( N_2 \) by either writing \( \Sigma F_z = 0 \) and \( N_1 \); or by choosing the pivot at the hands and using again \( \Sigma F_z = 0 \).
1. What is the tension in the erector muscle?  
   **Hint:** Align your $x$-axis with the axis of the spine.

2. A force from the pelvic girdle acts on the base of the spine. What is the component of this force in the direction of the spine? (This large force is the cause of many back injuries.)
Two 2.0 kg blocks on a level, frictionless surface are connected by a spring with spring constant 600 N/m, as shown in the figure. The left block is pushed by a horizontal force. At $t=0$ s, both blocks have velocity 3.0 m/s to the right. For the next second, the spring’s compression is a constant 1.0 cm.

1. What is the velocity of the right block at $t=1.0$ s?
2. What is the magnitude of $F$ during that 1.0 s interval?