Classical Physics II

PHY132
Lecture 35
Interference in thin films
Interference and Phase Difference

Interference is the destructive or constructive addition of "displacements" by waves from two or more sources at a spatial point ...

Interference is a simple consequence of the SUPERPOSITION PRINCIPLE:
- "displacements" caused by individual waves at any point simply ADD together
- it is a consequence of the linear character of the wave equation ...
Thus, a trough from one wave may coincide with an equally high peak of another, and the result may be no "displacement" at all...

(for minima: $\Delta \phi = n(2\pi) + \pi$, $n=0, \pm 1, \pm 2, \ldots$; i.e. an odd number of $\pi$);

Generally, 2 waves with equal $E_m$ and equal $f$ arriving at $x$ with phase diff. $\Delta \phi$:

$E(x,t) = E_m \cos(kx - \omega t) + E_m \cos(kx - \omega t + \frac{\Delta \phi}{2})$

$= 2E_m \cos \left( \frac{(kx-\omega t) + (kx-\omega t + \Delta \phi)}{2} \right) \cos \left( \frac{(kx-\omega t) - (kx-\omega t + \Delta \phi)}{2} \right)

= 2E_m \cos \left( kx - \omega t + \frac{\Delta \phi}{2} \right) \cos \left( - \frac{\Delta \phi}{2} \right)

= 2E_m \cos(kx - \omega t) \cos \left( \frac{\Delta \phi}{2} \right) = \left[ 2E_m \cos \left( \frac{\Delta \phi}{2} \right) \right] \cos(kx - \omega t)$

- the phase difference $\Delta \phi$ is a function of the path difference between either source and the point at $x$. It only depends on the position of the point.
Examples

Two-Slit Interference:

Two narrow slits are separated by a distance \( d = 0.300 \) mm. Normal incidence monochromatic light with \( \lambda = 660 \) nm falls on the slits, and forms an interference pattern on a screen \( L = 5.00 \) m away.

Calculate the distance between successive interference maxima in the intensity;

- **Maxima at positions** \( y_m \) **on the screen**:
  \[
y_m = L \tan \theta_m \approx L \sin \theta_m = mL \lambda / d = m(11.0 \) mm)

- **Note**: the small-angle approximation \( \sin \theta \approx \tan \theta \approx \theta \) may used because \( y_m \ll L \)
Phasor Notation - Intensity Profile

The intensity at a point \( P \) is proportional to the average electric field squared in \( P \):

\[ I_P \propto E_P^2 = (E_{A,P} + E_{B,P})^2, \]

- where the electric fields from \( A \) and \( B \) in \( P \) have to be added with the proper phase difference \( \Delta \varphi = (2\pi/\lambda) \times BK \)

in such situations easiest to use a PHASOR notation for \( E_{A,P} \) and \( E_{B,P} \):

- both are represented as vectors that rotate with angular frequency \( \omega \);
- their PROJECTIONS on the \( x \)-axis are the actual values \( E_{A,P} \) and \( E_{B,P} \) ...

The phasors have about equal magnitude

- if the slits at \( A \) and \( B \) have equal dimensions,
- and if \( CP \approx L \gg AB = d \)

The sum \( E_P \) is the projection of the phasor sum \( E_P \):

\[
\begin{align*}
\overline{I_P} &= S_P = \frac{E_P^2(x,t)}{c\mu_0} \\
&= \frac{1}{c\mu_0} \left[ 4E_m^2 \cos^2 \left( \frac{\Delta \varphi}{2} \right) \right] \cos^2 \left( kx - \omega t + \frac{\Delta \varphi}{2} \right) = \frac{2E_m^2}{c\mu_0} \cos^2 \left( \frac{\Delta \varphi}{2} \right) = \overline{I_0} \cos^2 \left( \frac{\Delta \varphi}{2} \right)
\end{align*}
\]
Interference from Equidistant Slits

Instead of only two slits, we may construct a grating, i.e. a system of very many equidistant slits (examples are optical gratings, CD's, etc.)
- one great advantage is that the brightness of the transmitted (or reflected) light increases as the number of slits increases...
- another advantage is that the interference maxima become much more narrow, allowing the wavelength spectrum of an unknown light source to be unraveled...

For instance: looking at $N=8$ equidistant ($d$) slits, when the phase difference between waves from adjacent slits is 45°:

$\Delta \phi = \frac{2\pi}{N} = 45^\circ$: $(\approx \frac{2\pi}{\lambda} \times d \sin \theta)$
- putting the phasors head-to-toe,
- we see quickly that the result is zero:
- i.e. phasors 1 and 5 cancel,
  phasors 2 and 6 cancel, etc.

Phasor addition:

$\Delta \phi = 45^\circ$
Interference from 8 Slits

For $N=8$, other **intensity** minima occur, for phase differences $\Delta \phi = \pm 45^\circ, \pm 90^\circ, \pm 135^\circ, \pm 180^\circ, \ldots = m2\pi/N$

i.e. minima occur **not only for** $\pm 180^\circ$!

"best" maxima occur when $\Delta \phi = 0^\circ, \pm 360^\circ, \ldots$ : $I_{\text{max}} = N^2I_0 = 64I_0$;

smaller "secondary" maxima occur in-between minima,

- e.g. the maximum at $\Delta \phi = 72^\circ$ occurs when five phasors form a closed pentagon, with the remaining three forming its "diagonal": $I_{72^\circ} \approx 9I_0$; etc.

- for larger $N$ the secondary maxima become rapidly smaller

  - and only the primary maxima survive and **contain** all the light's intensity!

  - cfr: $N=2$!

- **Grating (very large $N$)** is a powerful analysis tool!
Example 1

Reflection of normal incidence light off a thin layer of oil ($n=1.45$) on water ($n=1.33$):

- ray $A$, under reflection off a material of larger index of refraction, undergoes a phase shift $\Delta \varphi = \pi$
  
  - otherwise $\Delta \varphi = 0$
  
  - identical to a wave on a rope that meets a more massive piece of the rope or a fixed end:
    - the reflected wave is inverted (i.e. a phase shift of $\pi$)
    - the electric field in particular: $E_{\text{refl},a} = E_{\text{inc},a} \frac{n_a - n_b}{n_a + n_b}$

- Refracted waves have zero phase shift...

**Q:** dominant color of the reflected light?

$maxima$ for: $\Delta \varphi_{AB} = \left(2\pi/\lambda_{\text{oil}}\right)(2t) - \pi = m2\pi$

$\Rightarrow \lambda_{\text{air},m} = n_{\text{oil}}\lambda_{\text{oil}} = 4n_{\text{oil}}t/(2m+1)$, $m=0,1,2,\ldots$

$\Rightarrow \lambda_{\text{air},0} = 2204$ nm (infrared; not visible)

$\lambda_{\text{air},1} = 735$ nm (near infrared; not visible)

$\lambda_{\text{air},2} = 441$ nm (violet-blue; visible)

$\lambda_{\text{air},3} = 315$ nm (near UV; not visible)

- at larger angles, larger-$\lambda$ colors dominate ...

- Only thin layers reflect colors selectively; thick layers reflect all colors (many maxima)
Example 2

Interference from a thin wedge-shaped volume of air between two glass plates:

we ONLY consider the rays that reflect from the upper and lower surfaces of the THIN air-wedge...

- Assume the wavelength of the light to be \( \lambda = 500 \text{ nm} = 0.50 \mu\text{m} \).
- We count 48 interference fringes over distance \( L \) (the last at \( L \))

calculate the height \( h \) of the wire.

  - First, note ray \( B \) reflects off glass \( (n_{\text{glass}} > n_{\text{air}}) \)
    \( \Rightarrow \) acquires \( \Delta \varphi = \pi \) on reflection;
  - NOT so for ray \( A \! \! \! \! \! \! \! \) !
    \( \Rightarrow \text{A DARK spot at 0} ! \)
  - Twice height \( h \) must be equivalent to \( 48\frac{1}{2} \lambda \) path difference:
    \( 2h = 48.5\lambda \Rightarrow h = 12.1 \mu\text{m} \)
The Michelson interferometer is a powerful instrument that utilizes interference to measure pathlength differences of a fraction of a wavelength in size...

- the split monochromatic beams are recombined
- and show an interference pattern on the screen.
- Slightly changing one of the distances $L$ by $\lambda/4$ makes a bright fringe become dark!

If the speed of light would be different in the two perpendicular directions, the interferometer would give a different pattern when rotated by 90 degrees!
Example 3

An evacuated tube of \( d=5.00 \text{ cm} \) length is placed in arm 2 of a Michelson Interferometer.

The interference pattern is noted.

A gas is slowly let into the tube and because the wavelength in the gas differs from the wavelength in vacuum, the fringes “shift”.

48 fringes are seen to pass by a given point on the screen until the tube is filled.

Calculate the index of refraction of the gas at the final pressure...

The phase shift between the situations with the tube evacuated and the tube filled is \( 48(2\pi) \)

\[
\Delta \phi_{\text{vacuum-gas}} = 48(2\pi) = 2\pi \left| \frac{2d}{\lambda_{\text{gas}}} - \frac{2d}{\lambda_{\text{vac}}} \right| = 2\pi \left| \frac{2d n_{\text{gas}}}{\lambda_{\text{vac}}} - \frac{2d}{\lambda_{\text{vac}}} \right| = \frac{4\pi d}{\lambda_{\text{vac}}} (n_{\text{gas}} - 1)
\]

\[
\Rightarrow \quad n_{\text{gas}} - 1 = 48 \frac{\lambda_{\text{vac}}}{2d} = 2.62 \times 10^{-4}
\]
Interference by Same-Frequency Waves

Interference of two synchronous equal-frequency, equal-amplitude sound sources, ignoring reflections from walls, floor, ceiling, etc...

Maxima (twice the amplitude)

Minima (dead spots)

Maximum in P: $|AP - BP| = n\lambda$, $n=0,1,2,...$

Minimum in P: $|AP - BP| = (n+\frac{1}{2})\lambda$