Classical Physics II

PHY132
Lecture 34
Interference II
Interference and Phase Difference

Interference is the destructive or constructive addition of “displacements” by waves from two or more sources at a spatial point...

Interference is a simple consequence of the SUPERPOSITION PRINCIPLE:

- “displacements” caused by individual waves at any point simply ADD together
- it is a consequence of the linear character of the wave equation...

Thus, a trough from one wave may coincide with an equally high peak of another, and the result may be no “displacement” at all...

(in this case: $\Delta \varphi = n(2\pi) + \pi$, $n=0,\pm1,\pm2,\ldots$; i.e. an odd number of $\pi$);

Generally, 2 waves with equal $E_m$ and equal $f$ arriving at $x$ with phase diff. $\Delta \varphi$:

$$E(x,t) = E_m \cos(kx - \omega t) + E_m \cos(kx - \omega t + \frac{\Delta \varphi}{2})$$

$$= 2E_m \cos\left(kx - \omega t + \frac{\Delta \varphi}{2}\right) \cos\left(kx - \omega t - \frac{\Delta \varphi}{2}\right)$$

- the phase difference $\Delta \varphi$ is a function of the path difference between either source and the point at $x$. It only depends on the position of the point.
Example: Interference

Interference of two coherent monochromatic (point) sources A and B:

- **coherent**: wave fronts in sync
- **phase difference in** $P$:
  $$\Delta \varphi = k \Delta x = \left( \frac{2\pi}{\lambda} \right) \times |BP-AP|; \text{ here:}$$
  - path diff: $|BP-AP| = 8\lambda - 6\lambda = 2\lambda = \Delta x$
  $$\Rightarrow \Delta \varphi = k \Delta x = \left( \frac{2\pi}{\lambda} \right) \times 2\lambda = 4\pi$$
  $$\Rightarrow \text{CONSTRUCTIVE interference}$$

maxima: $d \sin \theta_{n,\text{max}} = n\lambda$, $n=0,\pm 1,\pm 2,\ldots$

- path diff $|BQ-AQ| = |5-5.5|\lambda = 0.5\lambda = \Delta x$
  $$\Rightarrow \Delta \varphi = k \Delta x = \left( \frac{2\pi}{\lambda} \right) \times 0.5\lambda = \pi$$
  $$\Rightarrow \text{DESTRUCTIVE interference}$$

minima: $d \sin \theta_{n,\text{min}} = (n+\frac{1}{2})\lambda$, $n=0,\pm 1,\pm 2,\ldots$

- **Intensity pattern**: alternating bright and dark lines:
  $$I_p = I_0 \cos^2 \left( \frac{\Delta \varphi}{2} \right) = I_0 \cos^2 \left( \frac{(2\pi/\lambda) \times \Delta x}{2} \right) = I_0 \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right)$$
Example

Two antenna masts, 10 m apart, are emitting at 91.1 MHz. Find the angle with the bisector of the antenna baseline for which there is a first minimum in intensity...

minima for: \( d \sin \theta_n = \left( n + \frac{1}{2} \right) \lambda \) \( \Rightarrow \) \( \sin \theta_n = \pm \frac{1}{2} \frac{\lambda}{d}, \pm \frac{3}{2} \frac{\lambda}{d}, \pm \frac{5}{2} \frac{\lambda}{d}, \ldots \)

\( \Rightarrow \sin \theta = \pm \frac{\lambda}{2d} = \pm \frac{c}{f^2d} = \pm \frac{3 \times 10^8}{91.1 \times 10^6 \times 20} = \pm 0.165 \Rightarrow \theta = \pm 0.166 = \pm 9.49^\circ \)

- note the dependence on \( \theta \) and on \( \lambda \):
  
  • for different \( \lambda \) the minima and maxima will occur at different \( \theta \)!

Calculate the relative intensity at 4°:

\[
\frac{I_P}{I_0} = \cos^2 \left( \frac{\Delta \phi}{2} \right) = \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) = \cos^2 \left( \frac{\pi f d \sin 4^\circ}{c} \right) = 78.6\%
\]
Interference …

Note the change of the *angle* \( \theta \) of the lines of maxima (and minima) with the change in the distance \( d \) between the sources \( A \) and \( B \).
the intensity at a point \( P \) is proportional to the average electric field squared in \( P \): 
\[
I_p \propto E_p^2 = (E_{A,P} + E_{B,P})^2, 
\]
- where the electric fields from \( A \) and \( B \) in \( P \) have to be added with the proper phase difference \( \Delta \varphi = (2\pi/\lambda) \times BK \) 

in such situations easiest to use a PHASOR notation for \( E_{A,P} \) and \( E_{B,P} \):
- both are represented as vectors that rotate with angular frequency \( \omega \);
- their PROJECTIONS on the \( x \)-axis are the actual values \( E_{A,P} \) and \( E_{B,P} \)...

The phasors have about equal magnitude
- if the slits at \( A \) and \( B \) have equal dimensions,
- and if \( CP \approx L >> AB = d \)

The sum \( E_P \) is the projection of the phasor sum \( E_P \):
\[
\left| I_p \right| = S_p = \frac{E_p^2(x,t)}{c\mu_0} 
= \frac{1}{c\mu_0} \left[ 4E_m^2 \cos^2\left(\frac{\Delta \varphi}{2}\right) \right] \cos^2(kx - \omega t + \frac{\Delta \varphi}{2}) = \frac{2E_m^2}{c\mu_0} \cos^2\left(\frac{\Delta \varphi}{2}\right) = I_0 \cos^2\left(\frac{\Delta \varphi}{2}\right)
\]
Interference from Equidistant Slits

Instead of only two slits, we may construct a grating, i.e. a system of very many equidistant slits (examples are optical gratings, CD’s, etc.)

- one great advantage is that the brightness of the transmitted (or reflected) light increases as the number of slits increases...
- another advantage is that the interference maxima become much more narrow, allowing the wavelength spectrum of an unknown light source to be unraveled...

For instance: looking at \( N=8 \) equidistant \((d)\) slits, when the phase difference between waves from adjacent slits is \(45^\circ\): 

\[
\Delta \phi = \frac{2\pi}{N} = 45^\circ: \quad (= \frac{2\pi}{\lambda} \times d \sin \theta)
\]

- putting the phasors head-to-toe,
- we see quickly that the result is zero:
- i.e. phasors 1 and 5 cancel,
  phasors 2 and 6 cancel, etc.

Phasor addition:

\[
\Delta \phi=45^\circ
\]
Interference from 8 Slits

For \( N=8 \), other intensity minima occur, for phase differences \( \Delta \phi=\pm 45^\circ, \pm 90^\circ, \pm 135^\circ, \pm 180^\circ, \ldots = m2\pi/N \)

i.e. minima occur not only for \( \pm 180^\circ \)!

“best” maxima occur when \( \Delta \phi=0^\circ, \pm 360^\circ, \ldots \) : \( I_{\text{max}}=N^2I_0 = 64I_0 \);

smaller “secondary” maxima occur in-between minima,

• e.g. the maximum at \( \Delta \phi=72^\circ \) occurs when five phasors form a closed pentagon, with the remaining three forming its “diagonal”: \( I_{72^\circ} \approx 9I_0 \); etc.

- for larger \( N \) the secondary maxima become rapidly smaller

• and only the primary maxima survive and contain all the light’s intensity!

• cfr: \( N=2 \)!

- Grating (very large \( N \)) is a powerful analysis tool!