Classical Physics II

PHY132
Lecture 32
Polarization
Summary Mirrors/Lenses

Summary:
$R > 0$ for center-of-curvature on outgoing side;
$s$ & $s' > 0$ for real, else virtual

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2}{R}
\]

\[
m = -\frac{s'}{s} = \frac{f}{f-s}
\]

<table>
<thead>
<tr>
<th>lens/mirror type</th>
<th>$s &gt; 0$ (REAL object)</th>
<th>$s'$</th>
<th>$m = -s'/s$</th>
<th>image type</th>
</tr>
</thead>
<tbody>
<tr>
<td>converging/concave ($f &gt; 0$)</td>
<td>$0 &lt; f &lt; s$</td>
<td>$s' &gt; 0$</td>
<td>$-\infty &lt; m &lt; 0$</td>
<td>real, inverted</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; s &lt; f$</td>
<td>$s' &lt; -s$</td>
<td>$1 &lt; m &lt; \infty$</td>
<td>virtual, upright</td>
</tr>
<tr>
<td>diverging/convex ($f &lt; 0$)</td>
<td>$s &gt; 0$</td>
<td>$f &lt; s' &lt; 0$</td>
<td>$0 &lt; m &lt; 1$</td>
<td>virtual, upright</td>
</tr>
<tr>
<td>flat ($f = \infty$)</td>
<td>$s &gt; 0$</td>
<td>$s' = -s$</td>
<td>$m = 1$</td>
<td>virtual, upright</td>
</tr>
</tbody>
</table>
The Eye

The eye, is a variable-focus lens with color imaging-detection system, and a built-in diaphragm to regulate light collection...

- the normal healthy eye can provide a sharp color image on the retina for object distances between ~25 cm (the "near point" with relaxed lens; actual value varies with age) and infinity (the "far point" with a fully tensioned ciliary muscle).

- the far-sighted eye focuses well for far objects located between infinity and a near point FARTHER than 25 cm; it needs converging eye glasses to be able to focus down to 25 cm (e.g. to read a book: reading glasses).

- the near-sighted eye focuses well for near objects, but needs divergent eye glasses to be able to focus far-away objects beyond its far point (e.g. to drive).

- the “power” of eyeglasses is given by the $1/f$ value, with optician units “Diopter”: $1\ \text{D} = \text{m}^{-1}$
Simple Magnifier (Loupe)

the simple magnifier is used to view small objects by effectively bringing them closer than the near point at 25 cm; \( f << 25 \text{ cm} \)

the object here is imaged at \( s' = 25 \text{ cm} \gg f \), such the eye can focus on it:

- with the object at \( \sim 25 \text{ cm} \), its ANGULAR size would have been \( h/25 \text{ cm} \); now its effective angular size is \( h'/25 \text{ cm} \)

- ANGULAR magnification \( M \):

- Magnification \( m \):

(eye focus at near point)

\[
m = -\frac{s'}{s} = s' \left( \frac{1}{s'} - \frac{1}{f} \right)
\]

\[
= s' \left( \frac{f - s'}{fs'} \right) = \frac{f - s'}{f}
\]

\[
= 1 - \frac{s'}{f} \approx 1 + \frac{25 \text{ cm}}{f}
\]

- Note that for eye focusing at infinity: \( s = f, s' = -\infty \):

  - i.e. the relaxed eye can view the image; the LINEAR magnification \( m = \infty \),
  - but the ANGULAR magnification is still \( M = 25 \text{ cm}/f \) exactly!

- Typical magnifier: \( f = 2-6 \text{ cm} \)
The microscope acts like a regular converging lens (object lens or **objective**), with \( s_1 > f_1, |m| >> 1 \), followed by a simple magnifier (**ocular**).

- **Objective** forms a real (inverted) image
- which is viewed with magnifier:

\[
m_1 = -\frac{s_1'}{s_1} \approx -\frac{L}{f_1} \quad \text{\((L >> f_1)\)}
\]

- **Ocular**:
  - Angular magnification for relaxed eye:

\[
(s_2' = -\infty):
M_2 = \frac{25 \text{ cm}}{f_2}
\]

- **Total magnification**:

\[
mM = -\frac{s_1'}{s_1} \cdot \frac{25 \text{ cm}}{f_2} = -\frac{L - f_2}{f_1} \cdot \frac{25 \text{ cm}}{f_2} \approx -\frac{L \times 25 \text{ cm}}{f_1 f_2}
\]

- **Best magnification for small** \( f_1, f_2 \):
Compound Instruments: Telescope

The telescope looks at very distant objects \((s = \infty)\) with a converging lens (object lens or **objective**), with \(s'_1 = f_1\), followed by a simple magnifier (ocular).

- **objective**: \(s_1 = \infty \rightarrow s'_1 = f_1\)
- **ocular**: for relaxed eye: \(s'_2 = -\infty \rightarrow s_2 = f_2\)
- **Total Angular magnification is**:

\[
M = \frac{\frac{\theta_2}{\theta_1}}{\frac{h}{f_2}} = \frac{h}{f_2} \cdot \frac{f_1}{f_2} = -\frac{f_1}{f_2} \quad \text{(inverted image)}
\]

- i.e. largest magnification for \(f_1 \gg f_2\)
Polarization

• Generally, the electric (and magnetic) field vector field of the EM waves has components in all directions (e.g. $y$, $z$)
  - all transverse to the propagation direction ($x$),
  - these components typically have a non-zero phase differences ...

• An ideal POLARIZER (or: polarization filter) is a material passes only a specific direction (the POLARIZATION AXIS) of the electric field vector (thus the $B$ field also)

• EM waves are called POLARIZED when the electric (and magnetic) field vector
  - has a single, unique direction
  - which is $\perp$ to the direction of propagation)
Intensity and Polarization

Remember: traveling waves carry energy and momentum!

- Energy Flow rate per unit area \( S \):
  \[
  S(x,t) = |E \times B|/\mu_0; \quad B = E/c \Rightarrow S = E^2/(c\mu_0),
  \]
  or:
  \[
  S_{\text{avg}} = \frac{1}{2} E_m^2/(c\mu_0)
  \]
- Thus: the energy flow rate per unit area (INTENSITY) is proportional to the Electric field strength squared...

Consider polarized light illuminating a polarizer with a polarization axis making an angle \( \phi \) with the polarization direction of the light...

- Only the COMPONENT (---) // to the polarization axis passes: \( E_{\text{pass}} = E_{\text{inc}} \cos \phi \)
- the INTENSITY of light that passes:
  \[
  I_{\text{pass}}/I_{\text{inc}} = (E_{\text{inc}} \cos \phi)^2/E_{\text{inc}}^2 = \cos^2 \phi
  \]
- Thus, if unpolarized light, which – on average – contains equal amounts of // and \( \perp \) components, passes an ideal polarizer,
- only half the light intensity is passed:
  \[
  I_{\text{pass}}/I_{\text{inc}} = I_{\text{inc,} //}/I_{\text{inc}} = I_{\text{inc}} \cos^2 \phi / I_{\text{inc}} = \frac{1}{2}
  \]
Example

A beam of unpolarized light passes successively through two polarizers $\perp$ beam.

The incident light has intensity $I_0$.

- The two polarizer axes make an angle of 90° with each other.
  
  The intensity of the passing light is thus ZERO:
  
  $$I_2 = I_1 \times (\cos 90°)^2 = 0$$

- Now, insert a third polarizer (label $m$) in-between the two already there, with its axis under 45° with either of the others.
  
  • Miracle: some light passes through:
  
  $$I_2 = I_m \times (\cos 45°)^2 = [I_1 \times (\cos 45°)^2] \times (\cos 45°)^2$$
  
  $$= \frac{1}{2} I_0 \times (\cos 45°)^2 \times (\cos 45°)^2 = \frac{1}{8} I_0 > 0!$$
Polarization by Reflection

Empirically (derivable from EM wave propagation): initially unpolarized sunlight reflecting off water is partially polarized // to the water surface.

When the angle between the reflected and refracted light beams is exactly 90°,
- the reflected light is fully polarized // to the water surface;
- the refracted beam is enhanced in light polarized ⊥ surface

The angle of incidence for which this occurs is called the Brewster Angle:

\[ \sin \theta_B = \frac{n'}{n} = \frac{n'}{\sin(90° - \theta')} = \frac{n'}{\cos \theta'} \Rightarrow \tan \theta_B = \frac{n'}{n} \]

- for air-water: \( \theta_B = \arctan(n'/n) = 53.1° \)