Classical Physics II

PHY132
Lecture 30
Lenses
Summary Mirrors

Summary:

$R > 0$ for center of curvature on *outgoing* side (concave), else $R < 0$ (convex)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \approx \frac{2}{R}; \quad m = -\frac{s'}{s} = \frac{f}{f - s}$$

<table>
<thead>
<tr>
<th>Mirror type</th>
<th>$s &gt; 0$ (REAL object)</th>
<th>$f$ ($R \approx 2f$)</th>
<th>$s'$</th>
<th>$m = -s'/s = f/(f - s)$</th>
<th>Image type</th>
</tr>
</thead>
<tbody>
<tr>
<td>concave ($R &gt; 0$)</td>
<td>$0 &lt; f &lt; s$</td>
<td>$f &gt; 0$</td>
<td>$s' &gt; 0$</td>
<td>$-\infty &lt; m &lt; 0$</td>
<td>real, inverted</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; s &lt; f$</td>
<td>$f &gt; 0$</td>
<td>$s' &lt; -s$</td>
<td>$1 &lt; m &lt; \infty$</td>
<td>virtual, upright</td>
</tr>
<tr>
<td>convex ($R &lt; 0$)</td>
<td>$s &gt; 0$</td>
<td>$f &lt; 0$</td>
<td>$f &lt; s' &lt; 0$</td>
<td>$0 &lt; m &lt; 1$</td>
<td>virtual, upright</td>
</tr>
<tr>
<td>flat ($R = \infty$)</td>
<td>$s &gt; 0$</td>
<td>$f = \infty$</td>
<td>$s' = -s$</td>
<td>$m = 1$</td>
<td>virtual, upright</td>
</tr>
</tbody>
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Curved Interfaces

consider a spherical air-glass interface:

for small angles $\theta$

(close to optical axis):

- $n\theta = n'\theta'$ (Snell's Law)
- Various triangles: $h/s = \tan \theta \approx \theta$; $h'/s' = \tan \theta' \approx \theta'$; $h/(s+R) = h'/(s'-R)$;

Then, eliminate $\theta, \theta', h, h'$ from these 4 equations to find the relation between $s, s', n, n', and R$:

$$\frac{s + R}{h} = \frac{s' - R}{h'} \implies \frac{s + R}{s\theta} = \frac{s' - R}{s'\theta'} = \frac{s' - R}{s'n\theta/n'} \implies 1 + \frac{R}{s} = \frac{n'}{n} \left(1 - \frac{R}{s'}\right)$$

$$\Rightarrow n + \frac{n}{s}R = n'- \frac{n'}{s'}R \implies \left(\frac{n}{s} + \frac{n'}{s'}\right)R = n'- n \implies \frac{n}{s} + \frac{n'}{s'} = \frac{n'- n}{R}$$

- magnification: $m \equiv \frac{h'}{h} = -\frac{s'\theta'}{s\theta} = -\frac{s'}{s\theta} \frac{n\theta}{n'} = \frac{s'/n'}{s/n}$

sign convention: $R>0$ if the center of curvature is on the OUTGOING side of the interface!!
The Lens

is a transparent object with at least one curved interface.
It is, in fact, a sequence of two interfaces: e.g. air-glass, followed by a glass-air...

- in the "**Thin Lens Approximation**", we take the lens to be very thin compared to the object and image distances...
- calculation: the image formed by the first interface with \( R_1 \),
- forms the object (at \(-s'\) for a thin lens!) for the 2nd interface of \( R_2 \):

\[
\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{R_1},
\frac{n'}{s'} + \frac{n}{s} = \frac{n-n'}{R_2}
\]

\[
\Rightarrow \quad \frac{n}{s} + \frac{n}{s''} = (n'-n) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

- A glass \((n'=n)\) lens in air \((n=1)\):

\[
\frac{1}{s} + \frac{1}{s''} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \equiv \frac{1}{f}; \quad m \equiv \frac{h'}{h} = -\frac{s'}{s}
\]

- Note, that this term depends uniquely on **material and shape**, and behaves EXACTLY as the inverse of a focal distance \(f\)!!
Converging Lens: Example 1

A plano-convex lens has radius of curvature of 7.5 cm (e.g. \( R_1=\infty \), \( R_2=7.5 \) cm) and index of refraction \( n=1.5 \).

Calculate the image of an object placed at \( s=25 \) cm from the lens by i) geometrical construction, ii) calculation...

- **Construction**: first, the focal points:

  \[
  \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\
  = 0.5\left(\frac{1}{\infty} - \frac{1}{-7.5}\right) \\
  = +\frac{0.5}{7.5} \Rightarrow f = +15 \text{ cm}
  \]

  - **Note**: 3 construction lines: //, through \( O \), focus

- **Calculation**:

  \[
  \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{15} - \frac{1}{25} = +\frac{2}{75} \\
  \Rightarrow s' = +37.5 \text{ cm (real image!)}
  \]

  \[
  m = -\frac{s'}{s} = -\frac{37.5}{25} = -1.5 \text{ (inverted image!)}
  \]

  - **Note**: for \( s> f \): real inverted image; \(-\infty < m < 0\)

04/19/2010
Camera

The focal length of the converging camera lens is given in mm: \( f = 52 \text{ mm}, 33 \text{ mm}, \text{ or } f = 150 \text{ mm}, 300 \text{ mm} \) for telelenses.

- For typical photos, the object distance \( s >> f \):
  \[
  \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{fs} \approx \frac{s}{fs} = \frac{1}{f} \]

- Then: the distance between the film (the image plane) and the lens \( s' \) is close to the focal distance \( f \):

- the magnification is thus closely equal to:
  \[
  m = \frac{s'}{s} = \frac{f}{f-s} \approx \frac{f}{s} \]
  
  • (i.e. the larger \( f \), the more magnification)

- the diaphragm of diameter \( D \) limits the lens opening to regulate light; for smaller openings the aberration is smaller and the depth sharpness improves...
  
  • brightness of image: \( \propto f^{-2} \) and \( \propto D^2 \), i.e. \( \propto (D^2/f)^2 \)
  
  • and: “f-number”; f-number \( \equiv f/D = f/2, f/4, \ldots , f/16 \)