Classical Physics II

PHY132
Lecture 27
MIRRORS AND GLASS: REFLECTION AND REFRACTION ...
Electromagnetic Spectrum

Frequency and wavelength: \( f \lambda = c \)

The **visible** part of the EM spectrum is only small, from 400 nm < \( \lambda < 800 \) nm

- different wavelengths are seen as different COLORS
- from short \( \lambda \) to long:
  - **violet**, **blue**, **green**, **yellow**, **orange**, **red**

**larger \( \lambda \) (lower \( f \)):**
- **INFRARED**, followed by
- **MICROWAVES:**
  - 0.1 mm < \( \lambda < 1 \) m,
- **RADIO WAVES:** \( \lambda > 1 \) m:
  - **UHF**, **VHF**, **MW**, **LW**, **VLF**, ...

**shorter \( \lambda \) (higher \( f \)):**
- **ULTRAVIOLET:** 1 nm < \( \lambda < 400 \) nm,
- **SOFT/HARD X-RAY:** 1 pm < \( \lambda < 1 \) nm, and
- **GAMMA:** \( \lambda < 1 \) pm, where instead of wavelength one usually uses photon **energy** to denote the radiation
much older than the discovery of the wave nature of light.

empirical facts:
- between a source and a receiver light moves along straight lines (“rays”) in a given uniform material
- at interfaces (reflective, refractive) the direction of the ray changes in predictably:

angles: measured w.r.t. the normal $\mathbf{n}$ to the surface of reflection or to the interface of refraction

picture:
- the incident ray, the outgoing ray (reflected or refracted), and the normal to the surface/interface *all lie in the same plane*...

- for REFRACTION: $n_{\text{inc}} \sin \theta_{\text{inc}} = n_{\text{refr}} \sin \theta_{\text{refr}}$
- for REFLECTION: $n_{\text{inc}} = n_{\text{refl}} \Rightarrow \theta_{\text{inc}} = \theta_{\text{refl}}$
Ray Optics and EM Waves

• Because the propagation direction is \( \perp \mathbf{E} \) and \( \mathbf{B} \), the propagation is in straight lines, until an conductive, dielectric, or magnetic surface is met...

• at a conductive surface, the changing electric and magnetic fields induce the mobile charges to oscillate in a synchronous motion, and that motion in turn causes the emission of EM waves...

• the reflection is caused by the synchronously phased motions of mobile charges everywhere in the surface, which causes the emergence of an EM wave front which is an accurate reflection of the incident wave

• similarly, at a dielectric surface both reflection and refraction take place: the refracted wave propagates at smaller speed inside the dielectric, and thus has a direction different from the incident
rays are lines of propagation: in a uniform medium they are straight.

rays change direction at interfaces between different media.

wave fronts are loci of maxima or of equal phase in the (electric) field; they are therefore everywhere \( \perp \) to the ray, and spaced by \( \lambda \).

- e.g. PLANE WAVE:
  Energy Flow rate (Power/Area) is constant!

- SPHERICAL WAVE:
  Energy Flow rate (Power/Area) \( \propto 1/r^2 \)

the speed of light in a medium \( v_n \) is related to \( c \) by the “index of refraction” \( n \):

\[
n \equiv c/v_n = \sqrt{(K_e\varepsilon_0K_m\mu_0)/\varepsilon_0\mu_0} \approx \sqrt{K_e}
\]

\( (K_m=1 \text{ for all non-ferromagnetic materials}) \)
Snell’s Law

Consider now in more detail refraction of an incident wave on an (e.g. air-glass) interface:

- During the time interval $\Delta t$, the ray at $b$ travels a distance $\Delta s_{\text{air}}$, while the ray at $a$ travels $\Delta s_{\text{glass}}$ into the glass...
- The frequency of the light must the same in air as in glass... (the shaking of the charges in the glass follows the wave’s frequency...)
- The speed of the light inside the glass $v_{\text{glass}}$ is smaller than $v_{\text{air}}$:

$$v_{\text{glass}}/v_{\text{air}}=(c/n_{\text{glass}})/(c/n_{\text{air}})=n_{\text{air}}/n_{\text{glass}}$$

- Because $v=\lambda f$, the wavelength $\lambda_{\text{glass}}$ is smaller than $\lambda_{\text{air}}$:

$$\lambda_{\text{glass}}/\lambda_{\text{air}}=n_{\text{air}}/n_{\text{glass}}$$

\[
\begin{align*}
\sin \theta_{\text{inc}} &= \frac{\Delta s_{\text{air}}}{d} = \frac{v_{\text{air}}}{d} \Delta t \\
\sin \theta_{\text{refr}} &= \frac{\Delta s_{\text{glass}}}{d} = \frac{v_{\text{glass}}}{d} \Delta t
\end{align*}
\]

\[
\begin{align*}
\sin \theta_{\text{inc}} &= \frac{v_{\text{air}}}{n_{\text{glass}}} = \frac{n_{\text{glass}}}{n_{\text{air}}} = \frac{n_{\text{refr}}}{n_{\text{inc}}} \\
\Rightarrow n_{\text{inc}} \sin \theta_{\text{inc}} &= n_{\text{refr}} \sin \theta_{\text{refr}}
\end{align*}
\]
Reflection

Note, this also “explains” the law of reflection:

\[
\frac{n_{\text{inc}} \sin \theta_{\text{inc}}}{n_{\text{refl}} \sin \theta_{\text{refl}}} = \frac{\sin \theta_{\text{inc}}}{\sin \theta_{\text{refl}}} = \frac{v_{\text{air}}}{v_{\text{air}}} = 1 \quad \Rightarrow \quad \theta_{\text{inc}} = \theta_{\text{refl}}
\]

I’m 6 ft=72” tall and my eyes are at 66”; how high should I hang a full-view mirror and what must be its minimum size?

- Simple geometry \(\Rightarrow\) the top of the mirror must be halfway between my eyes and the top of my head (i.e. 3” above my eyes), and the bottom halfway between my eyes and my soles
  - i.e. the minimum height of the mirror is exactly half the height of my body = 36”.
  - Similarly, the minimum width of the mirror is half the width of my body...
  - INDEPENDENT of object distance \(s\)!
  - IMAGE is behind the mirror:
    - Object distance: \(s' = s\)
    - Magnification: \(m=1\)
Flat Mirrors

A Flat Mirror is the simplest optical element

Rules and definitions:

- **OBJECT DISTANCE**: \( s > 0 \)
  (positive if rays EMERGE from the object towards the mirror; almost always positive)

- image BEHIND the mirror \( \Rightarrow \)
  VIRTUAL image, IMAGE DISTANCE \( s' < 0 \)
  - From construction:
    - Mirror: \( s' = -s; \ h' = +h \)

- definitions:
  - image on **same** side as object: \( s' > 0 \) (real image)
  - image on **other** side
    - as object: \( s' < 0 \) (virtual image)

- **MAGNIFICATION**:
  - \( m = h'/h = -s'/s = +1 \) (flat mirror)
Total Internal Reflection

A curious phenomenon occurs when light impinges on an glass-air interface, i.e. when $n_{\text{inc}} > n_{\text{refr}}$:

- Snell’s Law: $\sin \theta_{\text{refr}} = \sin \theta_{\text{inc}} \frac{n_{\text{inc}}}{n_{\text{refr}}}$

- Increasing the angle of incidence, see the Figure below, such that:

  $\sin \theta_{\text{inc}} \frac{n_{\text{inc}}}{n_{\text{refr}}} \geq 1$ i.e. $1 > \sin \theta_{\text{inc}} \geq n_{\text{refr}}/n_{\text{inc}}$:
  
  • Clearly $\sin \theta_{\text{refr}}$ cannot become larger than 1
  
  • $\Rightarrow$ NO REFRACTION for: $\sin \theta_{\text{inc}} \geq n_{\text{refr}}/n_{\text{inc}} \equiv \sin \theta_{\text{critical}}$
  
  • and for $\theta_{\text{inc}} \geq \theta_{\text{critical}}$ ALL the light is REFLECTED

- e.g.:

  glass ($n=1.5$) $\rightarrow$ air ($n=1.0$):

  $\theta_{\text{critical}} = \arcsin(n_{\text{refr}}/n_{\text{inc}})$

  $= \arcsin(0.67) = 42^\circ$

Note: the thickness of the rays indicates their relative intensity.
Applications

the efficient transmission of light through fibers (fiber-optics) is a very important application of the phenomenon of TOTAL INTERNAL REFLECTION:

- as long as all incident angles remain over the critical angle, the light is efficiently “piped” down the fiber...
- Of course, some absorption may be present...
- glass fibers are thin, highly flexible, cheap, and immune to electrical interference!
- modulated light can carry lots of information (now >1 GBit/s)