Classical Physics II

PHY132
Lecture 26
Light, Radiation, and Quantum Physics
Maxwell’s Equations

Four equations together completely describe all the material covered so far (i.e. all of electromagnetism); they are called Maxwell’s Equations:

on the direct sources of E and B:
- **Gauss’ Law for E**: \( \oint E \cdot dA = Q_{\text{encl}} / \varepsilon_0 \) (our workhorse for the E-field)
- **Gauss’ Law for B**: \( \oint B \cdot dA = 0 \) (no magnetic monopoles)

on the deep connections between E and B:
- **Ampere’s Law**: \( \oint B \cdot dl = \mu_0 \left( i_{\text{charge}} + \varepsilon_0 \frac{d \Phi_E}{dt} \right)_{\text{encl.}} \) (with Maxwell’s \( i_D \))
- **Faraday’s Law**: \( \mathcal{E} = \oint E \cdot dl = - \frac{d \Phi_B}{dt} \) (with Lenz’ Law)

  • note: symmetry would be perfect if magnetic monopoles and currents of monopoles existed …

Maxwell showed that these 4 laws predict the existence of ELECTROMAGNETIC waves, propagating through vacuum, at the speed of light \( c = 1 / \sqrt{\varepsilon_0 \mu_0} \)

- This was revolutionary at the time: waves were only thought to propagate in a MEDIUM …
- Maxwell postulated the AETHER as the medium for EM waves… with very weird properties …
Energy Flow

Energy density in an electromagnetic wave:

\[ u = u_E + u_B = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} = \frac{\varepsilon_0 E^2}{2} + \frac{E^2}{2c^2 \mu_0} = \varepsilon_0 E^2 \]

- i.e. the energy densities of the electric and magnetic fields are equal!

the ENERGY FLOW \( \mathbf{S} \) is defined as:
- the amount of energy transported per unit time (i.e. POWER),
- per unit area in the direction of travel...

i.e. take the energy \( dU \) in a thin volume \( A dx \), with \( A \parallel x \), the direction of travel:

\[ dU = uA dx = uAcdt \]

\[ S = \frac{P}{A} = \frac{dU}{Adt} = uc = \varepsilon_0 c E^2 = \varepsilon_0 c^2 EB = \frac{EB}{\mu_0} \]

- defining the Energy Flow VECTOR (Poynting Vector) \( \mathbf{S} \):

\[ \mathbf{S} = \frac{E \times \mathbf{B}}{\mu_0}; \quad \text{and:} \quad \frac{E B}{\mu_0} = \frac{E_{\text{max}} B_{\text{max}} \cos^2(\ldots)}{\mu_0} = \frac{E_{\text{max}} B_{\text{max}}}{2 \mu_0} \]

- e.g. the laser’s Power/Area is:

\[ S = \frac{(1.2 \times 10^6 \ \text{V/m})(4.0 \times 10^{-3} \ \text{T})}{8\pi \times 10^{-7} \ \text{T} \cdot \text{m/A}} = 1.9 \times 10^9 \ \text{W/m}^2 = 1.9 \ \text{W/mm}^2 \]
Momentum Flow

The Momentum flow RATE (i.e. \( dp/\text{dt} \)) of the EM wave per unit transverse area is then:

\[
P = F v = \frac{dp}{dt} c ; \quad \frac{P}{A} = S ; \quad \Rightarrow \frac{dp}{dt} = \frac{P}{c} = \frac{SA}{c} \quad \Rightarrow \frac{1}{A} \frac{dp_{\text{EM}}}{dt} = \frac{S}{c}
\]

- If this momentum rate flow is fully absorbed, there will be a resulting pressure (i.e. average force per unit area), called radiation pressure \( P_{\text{rad}} \):

\[
P_{\text{rad, abs}} = \frac{F}{A} = \frac{1}{A} \frac{dp}{dt} = \frac{1}{A} \frac{dp_{\text{EM}}}{dt} = \frac{S}{c}
\]

- If, in contrast, the wave is fully reflected, the momentum-rate-of-change is twice as big because \( \Delta p = p_{\text{refl}} - p_{\text{inc}} = -2p_{\text{inc}} \), and:

\[
P_{\text{rad, refl.}} = \frac{F}{A} = \frac{1}{A} \frac{dp}{dt} = \frac{2}{A} \frac{dp_{\text{EM}}}{dt} = \frac{2S}{c}
\]
Electromagnetic Spectrum

Frequency and wavelength: \( f \lambda = c \)

The **visible** part of the EM spectrum is only small, from 400 nm < \( \lambda \) < 800 nm
- different wavelengths are seen as different COLORS
- from short \( \lambda \) to long:
  - **violet, blue, green, yellow, orange, red**

**larger \( \lambda \) (lower \( f \))**:
- **INFRARED**, followed by
- **MICROWAVES**:
  - 0.1 mm < \( \lambda \) < 1 m,
- **RADIO WAVES**: \( \lambda > 1 \) m:
  - UHF, VHF, MW, LW, VLF, ...

**shorter \( \lambda \) (higher \( f \))**:
- **ULTRAVIOLET**: 1 nm < \( \lambda \) < 400 nm,
- **SOFT/HARD X-RAY**: 1 pm < \( \lambda \) < 1 nm, and
- **GAMMA**: \( \lambda < 1 \) pm, where instead of wavelength one usually uses photon energy to denote the radiation
Blackbody Radiation

- England and fine china: how hot is a red-hot or white-hot oven?
- Can one predict the radiation spectrum from temperature?
- First attempts: blow-up on the short wavelength end. Refined attempts: close but not quite right.
That’s odd…

• Wien, 1896: semi-empirical fit that was pretty close …
• Max Planck: ties some aspects of Wien’s law to statistical thermodynamics.
• But not quite right! Heinrich Rubens of Universität Berlin found discrepancies with his own measurements …
• Oct. 7, 1900: Rubens pays a Sunday afternoon visit to Planck. By that evening, Planck has found an answer that works.

Sample of the Rubens–Kurlbaum data which led Planck to guess his radiation formula [R1]. $\rho$ is plotted versus $T$ for $\lambda = 51.2\mu m$. (“berechnet nach” means “computed after”, “beobachtet” means “observed.”) The curves marked “Wien” and “Lord Rayleigh” refer to best fits to the Eqs. (19.5), (19.17), respectively. The curves marked “Thiesen” and “Lummer–Jahnke” refer to theoretical proposals which are not discussed in this book. Planck’s formula is not yet plotted.

From: A. Pais, “Subtle is the Lord”
Planck’s theory

- Imagine a black body made of a conductor (metal):
  Electromagnetic standing waves must have nodes at boundaries.
- **Associate energy** of each wave with its **frequency**: \( E = hf \).
  Wavelength: \( E = h\nu \)
  \( h\nu = 1240 \text{ eV}\cdot\text{nm} \)
- Along with established statistical thermodynamics, it provides a good fit!
- But quite a radical solution!
  *Energy is quantized!*
- *Is emitted and absorbed in discrete amounts*

Max Planck
(1858-1947).
Nobel Prize: 1918
The difference is subtle…

In fact, some people don’t even notice!

- Lord Rayleigh (John William Strutt) proposes corrections to Wien’s law in 1900
- James Hopwood Jeans contributes further in 1905!
- Will anyone take Planck seriously?
Einstein’s revolution #2

- Take Planck at his word: \( E = hf \)
- Special relativity for light (no mass) then gives
- And thus **photons** carry momentum

\[
E = hf = pc
\]

- Explains the photoelectric effect: must reach a threshold **wavelength** to knock electrons off a metal surface in vacuum:

\[
p = \frac{h}{\lambda}
\]

- What Einstein got the Nobel Prize for!