Classical Physics II

PHY132
Lecture 18
Faraday’s Law, Lenz’ Law
Varying Magnetic Flux

- The AMAZING conclusion from the above discussion is that (between the plates) a magnetic field is generated by a time-varying electric flux $d\cdot E/\text{dt}$!

- We will now discuss the mirror effect: a time-varying magnetic flux $d\cdot B/\text{dt}$ induces an electric field!

- Magnetic Flux defined: $\cdot B \cdot \cdot B \cdot dA$
  - Note Gauss' Law for Magnetism: $\oint B \cdot dA = 0$

- Consider a conducting wire of length $L$ moving with a velocity $v$ perpendicular to a uniform magnetic field $B$: 
Recap the definition of Magnetic Flux:

\[ \mathbf{B} \cdot d\mathbf{A} \]

- Note: for a CLOSED surface:

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

Gauss' Law for Magnetism; absence of magnetic monopoles

Now consider a \( L \) m long piece of conducting rod moving with velocity \( \mathbf{v} \) perpendicular to a uniform and constant magnetic field \( \mathbf{B} \):

the Lorentz force on charge carriers \( q \) in the rod is:

\[
\mathbf{F}_L = q \mathbf{v} \times \mathbf{B} (-\mathbf{k}) = q \mathbf{v} 0 0 = -\mathbf{j} (-qvB) = qvB\mathbf{j}
\]

- the \( \mathbf{F}_L \) pushes charges ... 
- to accumulate at the ends of the rod 
- an EMF is created:

\[
\mathcal{E} = \Delta V = -\int_{0}^{L} \mathbf{E} \cdot d\mathbf{l} = -\frac{\mathbf{F}_L}{q} L = -vB L
\]

\[
= -BL \frac{dx}{dt} = -B \frac{dA}{dt} = -\left. \frac{d\Phi_B}{dt} \right|_{dt}
\]
Experiments (Faraday et al.) have shown that:

**CHANGES IN MAGNETIC FLUX** $\Phi_B$ over a surface *(non closed!)*

cause an **INDUCED EMF** in the curve bounding the surface

**Faraday’s Law:**

$$\mathcal{E} = \oint_{\text{bounding curve } C} \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d}{dt} \int_{\text{surface bounded by curve } C} \mathbf{B} \cdot d\mathbf{A} = -\frac{d}{dt} \int_{C} B \, dA \cos \theta_{dA,B}$$

Thus, a changing magnetic flux can result from:

- a change in field $B$: i.e. $dB/dt \cdot 0$
- a change in area $A$: i.e. $dA/dt \cdot 0$
- and/or a change in the angle between $B$ and $A$: i.e. $d\cos \theta_{A,B}/dt \cdot 0$

We’ll consider examples of each of these in the following...
Changing Magnetic Field \( \rightarrow \) EMF

Consider a loop of \( N \) windings and area \( A \) immersed in a uniform magnetic field \( B \), making an angle of 30° with the normal to the plane of the loop.

Assume the field is changing over time as: \( B = B_0 e^{-t/\tau} \) (e.g. due to a charging capacitor) with • some characteristic time constant:

- Note the field is DECREASING with time:
  \[
  \frac{dB}{dt} = -(B_0/\tau) e^{-t/\tau} \quad \text{•} \quad 0
  \]
  and \( \mathcal{E} = -d\cdot B/dt = -[B_0NA\cos(30 \degree)/\tau] e^{-t/\tau} \)

- At \( t=0 \): \( \mathcal{E}_0 = -d\cdot B/dt|_{t=0} = -B_0NA\cos(30 \degree) /\tau \)
- At \( t=\tau \): \( \mathcal{E}_0 = -d\cdot B/dt|_{t=\tau} = 0 \)

The **meaning of the -sign in** \( \mathcal{E}=-d\cdot B/dt \):

- the induced EMF is such that it **opposes the change in flux**;
  - i.e. the resulting **current** in the loop will be \( I_{\text{induced}} = \mathcal{E}/R \),
  - and the induced current will flow in **such a direction** that it **opposes** the change in flux (a decrease for this example),
- i.e. here it will be such that \( B_{\text{induced}} \text{ strengthens} \) the instantaneous field which is decreasing with time...

This is **Lenz' Law**
Using Lenz’ Law, determine the direction of the induced current in circuit 2 (a → b or b → a) when:

- with switch S closed, coil 2 is moved CLOSER to coil 1:
  coil 2 moves into a **stronger** \( B \) field (\( B \) to right) \( \Rightarrow \) flux in coil 2 *increases*.  
  Lenz’ Law: current induced in 2 *opposes* this \( \Rightarrow \) induced field to the *left*, i.e. induced current in 2: \( a \rightarrow b \)

- keeping the switch closed, resistance \( R \) is *reduced*:
  Field (to *right*) from coil 1 becomes *stronger* \( \Rightarrow \) flux in coil 2 *increases*.  
  Lenz’ Law: current induced in 2 *opposes* this \( \Rightarrow \) induced field to the *left*, i.e. induced current in 2: \( a \rightarrow b \)

- after having been closed, switch S is now *opened*:
  when switch opens: \( B \) field (\( B \) to *right*) suddenly *decreases* \( \Rightarrow \) flux in 2 *decreases*.  
  Lenz’ Law: current induced in 2 *opposes* this \( \Rightarrow \) induced field to the *right*, i.e. induced current in 2: \( b \rightarrow a \)
Changing the Flux Area A

The slide-wire generator:
force $F_{\text{ext}}$ pulls the rod with constant $v$:

- the area of the loop changes (increases):
  \[ \frac{dA}{dt} = L \frac{dx}{dt} \]
- the Lorentz force $F_L$ pushes the charge carriers up in the rod,
- from where they flow around the loop to the bottom of the rod:
  \[ |\mathcal{E}| = d \cdot \frac{B}{dt} = B \frac{dA}{dt} = BLv \]

the flux $\cdot B = B_{\text{ext}} \cdot dA$ is INcreasing in this example; indeed the induced current $I_{\text{ind}} = \mathcal{E}/R = BLv/R$ produces a $B_{\text{induced}}$ OUT of the page, in accordance with Lenz' Law!

- Note that the induced current interacts with $B_{\text{ext}}$ to give a force to the LEFT, opposite and equal to $F_{\text{ext}}$ (if not, we would get free energy!)

- Dissipated power: $P_{\text{dis}} = I^2R = B^2L^2v^2/R$
- Applied power: $P_{\text{ext}} = F_{\text{ext}} \cdot v = -F_{\text{opposing}}v = I_{\text{ind}}(\bullet)LB_{\text{ext}}v = B^2L^2v^2/R$

$\Rightarrow$ Lenz' Law ensures Energy Conservation...
Changing the Angle between $\mathbf{A}$, $\mathbf{B}$

Consider a static and uniform $\mathbf{B}$-field to the right. $\mathbf{A}$ wire loop ($N$ windings, area $\mathbf{A}$) rotates around an axis perpendicular to $\mathbf{B}$ (perpendicular to the paper), with constant angular velocity $\omega$:

- Thus only the **relative orientation** of $\mathbf{B}$ and $\mathbf{A}$ varies with time...
- The change in flux:

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(\mathbf{B} \cdot \mathbf{A}) = BA \frac{d\cos\theta_{AB}}{dt} = BA \frac{d\cos(\omega t)}{dt} = -\omega BA \sin(\omega t)$$

$$\Rightarrow \mathcal{E} = -N \frac{d\Phi_B}{dt} = \omega NBA \sin(\omega t)$$

- i.e. induced EMF varies with time sinusoidally with amplitude $\cdot NBA$
- Numerical example: an **AC generator**:

$$f = 60 \text{ Hz} = \frac{\omega}{2\pi}, \ N = 500, \ A = 3.0 \text{ m}^2, \ B = 0.10 \text{ T}$$

$$\Rightarrow \mathcal{E} = 2f \cdot NBA = 120 \cdot 500 \cdot 3.0 \cdot 0.10 = 57 \text{ kV},$$

i.e. good for High Voltage power line distribution