### Formulae:

#### Electric Charge $Q$ [C]:

\[ Q_e = -1.602 \times 10^{-19} \text{ C} = -e, \quad Q_p = +e, \quad m_e = 9.11 \times 10^{-31} \text{ kg}, \quad m_p = 1.67 \times 10^{-27} \text{ kg} \]

**Current:** $I \equiv \frac{dQ}{dt}$  
**Unit:** Ampere $A \equiv \text{C/s}$

\[ I = q n \nu \delta \propto E \]

$q$ is the charge of the mobile charges, $n_q$ their density, $A$ the wire’s cross section, $E$ the local electric field strength

#### Coulomb Force:

Force between two point-charges $Q_1$ and $Q_2$ separated by distance $r_{12}$

\[ F_{1 \rightarrow 2} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 r_{12}^2} \hat{r}, \quad F_{2 \rightarrow 1} = \frac{Q_2 Q_1}{4\pi \varepsilon_0 r_{12}^2} \hat{r} \]

\[ F_1 = 8.998 \times 10^9 \text{ N/m}^2/\text{C}^2, \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{Nm}^2) \]

#### Electric Field (Vector Field!): $E \equiv \mathbf{F}_E/q$

- **Field of a point charge $Q$**
- **Field outside a spherically symmetric charge $Q$ ($r>R$)**
- **Field of a line charge $\lambda=Q/L$**
- **Field outside ($r_n>R$) a cylindrically symmetric charge $\lambda$**
- **Field of a planar charge $\sigma=Q/A$**; for infinite plane, or close compared to the size of the plane...
- **Field of a uniformly charged ring, radius $R$, on a line $\perp$ ring and through the center ($\equiv x$-axis!)**

**Field inside a conductor**: $E=0$ when no EMF is present: the mobile charges rearrange themselves to cancel an external field ($E\neq0$ when an EMF is present!)

#### Energy density $u$ (energy/unit volume) of electric field $E$:

\[ u = \varepsilon_0 E^2/2 \]

#### Gauss’ Law for $E$:

**Flux** $\Phi_E \equiv \oint_{\text{closed surface}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$

Choose the Gaussian surface to match the symmetry of charge distribution and $E$-field!

#### Electric Potential (Scalar Field!):

\[ \Delta V = V_b - V_a = -\int_a^b \mathbf{E}_{\text{static}} \cdot d\mathbf{l} = 0; \quad \mathbf{E}(r) = -\frac{\nabla V(r)}{\nabla} = \left( -\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right) \]

- **Potential of a point charge $Q$ ($V(\infty) \equiv 0$)** or outside a spherically symmetric charge $Q$ ($r>R$)

\[ V(r) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r} \quad \Rightarrow \quad V = qV \]

- **Potential of a line charge $\lambda=Q/L$** or outside a cylindrically symmetric charge $\lambda=Q/L$ ($r_n>R$) (choice: $V(R) \equiv 0$)

\[ V = -\frac{\lambda}{2\pi \varepsilon_0} \ln \frac{r_n}{R} \quad (r_n \text{ is distance normal to the line}) \]

- **Potential of a plane charge $\sigma=Q/A$ ($V(0) \equiv 0$); or of an infinite plane; or closeby, compared to the size of the plane**

\[ V(x) = -\frac{\sigma}{2 \varepsilon_0} x \quad (x \text{ is normal to the plane}) \]

- **Potential INSIDE a conductor** $V = \text{constant because inside } E=0$

#### Electric Dipole: dipole moment $p \equiv Qd$

\[ (d=\text{distance vector from the } -Q \text{ to the } +Q \text{ charge}) \]

**Potential energy** of dipole in field $E$:

\[ U = -p \cdot E \quad \text{(choice: } U=0 \text{ when } p \perp E) \]

#### ElectroMotive Force (emf) $\mathcal{E} \equiv \frac{1}{q} \int \mathbf{F}_{\text{ext}} \cdot d\mathbf{l}$; $\mathbf{F}_{\text{ext}}$ from chemistry, mechanics, etc.

**Unit:** Volt $V \equiv J/C$
<table>
<thead>
<tr>
<th><strong>Resistance:</strong> $R \equiv dV/dI \approx V/I$ (Ohm’s Law)</th>
<th><strong>Unit:</strong> Ohm $\Omega \equiv V/A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conducting wire of cross section $A$ and length $L$</td>
<td>$R = \rho L/A$; resistivity $\rho$ is material-specific</td>
</tr>
<tr>
<td>Resistors in <strong>Series:</strong> $R_{\text{tot}} = R_1 + R_2 + \ldots$</td>
<td>Resistors in <strong>Parallel:</strong> $R_{\text{tot}}^{-1} = R_1^{-1} + R_2^{-1} + \ldots$</td>
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<tr>
<td>Power dissipation in a resistor:</td>
<td>$P = -dU/dt = I(t) \cdot V_R(t) = I_0^2 R$</td>
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<thead>
<tr>
<th><strong>Capacitance:</strong> $C \equiv Q/V$</th>
<th><strong>Unit:</strong> Farad $F \equiv C/V$</th>
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<tbody>
<tr>
<td>Capacitance of a conducting sphere of radius $R$:</td>
<td>$C = 4\pi \varepsilon_0 R$</td>
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<tr>
<td>Capacitance of two parallel plates, of area $A$ and separated by $d$, and filled with dielectric $K$:</td>
<td>$C = K \varepsilon_0 A/d$; $K \equiv$ (relative) dielectric constant</td>
</tr>
<tr>
<td>Capacitors in <strong>Parallel:</strong> $C_{\text{tot}} = C_1 + C_2 + \ldots$</td>
<td>Capacitors in <strong>Series:</strong> $C_{\text{tot}}^{-1} = C_1^{-1} + C_2^{-1} + \ldots$</td>
</tr>
<tr>
<td>Energy stored in capacitor $C$ carrying charge $Q$:</td>
<td>$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} Q_0^2/C$</td>
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<tr>
<th><strong>Kirchhoff’s Rules:</strong></th>
<th><strong>Rule #1:</strong> For any circuit node: the sum of all incoming (+ve) currents plus all outgoing (-ve) currents is zero: $\sum I_i = 0$</th>
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<tr>
<td></td>
<td><strong>Rule #2:</strong> Going through any closed circuit loop the sum of potential difference must be zero (beware of the signs!): $\sum_{\text{closed loop, } i} \Delta V_i = 0$</td>
</tr>
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| **RC Circuit:** $q(t) = q_0 e^{-t/\tau}$ | $q_0$ is the initial charge on the capacitor at $t=0$; characteristic time $\tau \equiv RC$ |
| **$\varepsilon RC$ Circuit:** $q(t) = q_\infty (1 - e^{-t/\tau})$ | $q_\infty = C\varepsilon$ is the ultimate charge at $t=\infty$; characteristic time $\tau \equiv RC$ |