Classical Physics I

PHY131
Lecture 6
Friction Forces and Newton’s Laws

Please set your Clicker to Channel 21
Recap

• Newton’s Laws:
  - 1 & 2: $F_{Net} = ma$
    - LHS: All the forces acting ON the object of mass $m$
    - RHS: the resulting acceleration, from which the motion can be deduced
  - 3: $F_{AB} = -F_{BA}$ (“Action is Reaction”)

• Valid only in Inertial Reference Frames!

• Use Free Body Diagram to find the Net Force $F_{Net}$:
  - diagram with all the (vector) forces acting ON THE BODY OF MASS $m$
  - Put the axis convention you will be using on it as well
  - Make separate diagrams for separate parts (if any) of the full problem...
Frictional Forces

- Generally, friction always opposes (relative) motion between the materials in question (block and incline, ball and air, fish and water, etc...). \( F_f \) is directed anti-parallel to the relative motion (if any).
  - In case of fluids and gases we speak of "Drag"; we’ll discuss this mostly later...
  - in case of rubbing surfaces we speak of "Friction"; today's topic
- Friction has "atomic causes", essentially electric in nature, and is intimately related to the detailed surface properties and smoothness of the rubbing surfaces... Note, that "smooth" often means more rather than less friction!
- Empirical:
  - Moving a Crate: Initially, a certain force is needed to start the crate moving; to "un-glue" the crate, one has to overcome Static Friction, which is typically larger than the force needed to keep it moving at constant velocity - Kinetic Friction...
  - In case of no motion, the static frictional force counteracts exactly the (increasing) external force trying to move the crate, until the external force overcomes static friction...
    Static friction has thus no fixed value, but only a maximum!
  - The (maximum) value of the friction forces is - in most cases - closely proportional to the Normal force \( N \) that presses the rubbing surfaces - crate bottom and floor - together:
- \( F_f \approx \mu N \), where \( \mu = \mu_s \) for static friction is typically larger than \( \mu = \mu_k \) for kinetic (= moving) friction...
Friction – 1st Example

- block of mass $M = 2.0 \text{ kg}$ sits on a horizontal table. The coefficients of static and kinetic friction are $\mu_s = 0.30$ and $\mu_k = 0.25$ respectively.
  - Calculate the minimum horizontal force $F$ necessary to start the block moving.
    - the minimum force needed is to just overcome static friction:
      \[
      \text{W} + \text{N} = ma, j = 0; \text{N} = -\text{W}; \quad F + F_{fs} = 0; \quad F_{fs} = \mu_s N(-i) = -0.30 \times 2.0 \times 9.80 \text{ i} = -5.9 \text{ i} \text{ N}
      \]
    - Calculate the acceleration of the block afterwards:
      \[
      F + F_{fk} = ma = (\mu_s - \mu_k)mg \text{ i} = 0.98 \text{ i} \text{ N}; \quad a = 0.48 \text{ m/s}^2
      \]
Friction – 1st Example

- For what angle is $F$ minimal?

- The weight $W$ of the block is now partially counteracted by the upward component of $F$.

- Because the vertical acceleration is zero, we have:
  
  $$-W + N + F\sin\theta = 0,$$
  
  and
  
  $$N = W - F\sin\theta$$

- Thus, the minimum force $F\cos\theta$ is needed to just overcome static friction:
  
  $$F\cos\theta = F_f = \mu_s N = \mu_s (W - F\sin\theta) \Rightarrow F = \mu_s W/(\cos\theta + \mu_s\sin\theta) = F(\theta)$$

  - The minimum occurs when $dF/d\theta = 0$:
    
    $$dF/d\theta = \mu_s W(-\sin\theta + \mu_s\cos\theta)/(\cos\theta + \mu_s\sin\theta)^2$$
    
    which equals zero for $\tan\theta = \mu_s$, i.e. $\theta = 16.7^\circ$;

  - The force $F$ is then:
    
    $$F = \mu_s W/(\cos\theta + \mu_s\sin\theta) = \sin\theta W = 5.6 \text{ N} \ (<5.9 \text{ N we found before!})$$
Block on Slope

- Single block of mass \( M = 3.0 \text{ kg} \), is initially at rest on a slope of angle \( \theta \). The angle is slowly increased until the block starts sliding at \( \theta = 25^\circ \)

  - Q1: calculate the coefficient of static friction...

- The block is at rest in the direction perpendicular to the slope, and thus \( F_{\text{Net,\perp}} = F_y = N - W \cos \theta = 0 \).

- When the block starts sliding: \( F_{fs} = W \sin \theta \). Thus, for the parallel forces: \( F_{\text{Net,\parallel}} = F_x = W \sin \theta - F_{fs} = 0 \).
  \[ F_{fs} = \mu_s N = \mu_s W \cos \theta = W \sin \theta \]
  - Thus: \( \mu_s = \tan(25^\circ) = 0.47 \) (independent of mass \( M \)!)
Block on Slope

- Q2: With $\theta$ kept at 25° the block slides down the slope (with constant acceleration) and slides a distance of 1.60 m in 2.0 seconds. Calculate the coefficient of kinetic friction

- when the block began sliding:

$$F_{Net} = W\sin\theta - F_{fk}$$
$$= W\sin\theta - \mu_k N$$
$$= W\sin\theta - \mu_k W\cos\theta$$
$$= mg (\sin\theta - \mu_k \cos\theta)$$
$$= ma$$

$\Rightarrow \mu_k = \tan\theta - a/(g\cos\theta)$

- Thus, we need the constant acceleration $a$ from the rest of the info:

$$D = \frac{1}{2} at^2 \Rightarrow a = 2D/t^2 = 0.80 \text{ m/s}^2$$

Then: $\mu_k = \tan\theta - a/(g\cos\theta) = 0.38$
A braking car…

- A 60 mi/hr car is braking with wheels blocked over a wet road. The coefficient of kinetic friction between tires and road is $\mu_k=0.40$. Ignore drag.

- **Calculate the braking distance:**
  - $F_{net} = F_k = \mu_k N = \mu_k mg = ma \Rightarrow a = \mu_k g$
    - in direction opposite to $v_0$
  - $v^2 - v_0^2 = 2aD \Rightarrow D = \frac{v_0^2}{2a} = \frac{v_0^2}{2\mu_k g}$
Classical Physics I

PHY131
Lecture 8
Circular Motion: kinematics
Recap: Friction

- Generally, friction always opposes (relative) motion between the materials in question (block and incline, ball and air, fish and water, etc...).
  \( \mathbf{F}_f \) is directed anti-parallel to the motion (if any).
  - In case of fluids and gases we speak of "Drag"; we’ll discuss this mostly later...
  - in case of rubbing surfaces we speak of "Friction"

- Friction has "(sub)microscopic causes", essentially electric in nature, and is intimately related to the detailed surface properties and smoothness of the rubbing surfaces... Note, that "smooth" often means more rather than less friction!

- Empirical:
  - \( \mathbf{F}_f \approx \mu \mathbf{N} \), where \( \mu = \mu_s \) for static friction is typically larger than \( \mu = \mu_k \) for kinetic (= moving) friction...
  - The direction of friction is opposing the parallel force or the motion...
  - The normal force \( \mathbf{N} \) presses the surfaces together; it is normal, i.e. perpendicular, to the surfaces.
Example

• Two blocks on a horizontal table have masses $M$ and $m$, and are linked by a (massless) rope. The coefficients of static and kinetic friction are 0.30 and 0.20 respectively, and the same for both blocks. The first mass $m$ is pulled at by an horizontal force $F$.
  
  - Q1: what is the minimum value of $F$ to get the blocks moving?
  - Q2: what is the acceleration that the blocks undergo when that force is applied?
  - Q3: What can you say about the tension $T$ in the rope in that case?

  • Two blocks: separate into TWO diagrams, linked by rope...
Example

- Two blocks: separate into TWO diagrams, linked by rope, i.e. they have a common motion, velocity $v$, acceleration $a$...
  - $F - T - F_{fk} = ma$
  - $F - T - \mu_k mg = ma$

- $T - F_{fk} = Ma$
  - $T - \mu_k Mg = Ma$

- **Q1**: $F_{min} = ?$
  1. $F - T - \mu mg = ma$
  2. $T - \mu Mg = Ma$
  $$F = (M + m)(\mu g + a) \Rightarrow F_{min} = (M + m)\mu_s g$$

- **Q2**: $a$ with $F = F_{min}$?
  $$F = F_{min} = (M + m)(\mu_k g + a) \Rightarrow a = \frac{F_{min}}{M + m} - \mu_k g = (\mu_s - \mu_k)g$$

- **Q3**: $T = ?$
  $$T = M(a + \mu_k g) = M\left((\mu_s - \mu_k)g + \mu_k g\right) = \mu_s Mg$$
Example of Two-Part Problem

- A block of mass $M=2.0 \text{ kg}$ lies on an incline (slope) that makes an angle $\theta=30^\circ$ with the horizontal. A massless rope connects $M$ to a block of mass $m=1.5 \text{ kg}$, which is suspended over a (frictionless, massless) pulley at the top of the incline. The magnitude of the friction between $M$ and the incline is $F_f=3.0 \text{ N}$.

- Calculate the acceleration of the masses.
  - Two blocks connected by rope: separate into **TWO diagrams**...
  - Note: if one block goes up, the other must go down - thus, choose the positive "$x$"-direction (and of $v$ and $a$) **CONSISTENTLY**:

  - Note: We don’t know direction of $F_f$ until we know the direction of motion! Find out with $F_f=0$ first!

  - Two equations, two unknowns: $T$ and $a$; Solve!

\[
\begin{align*}
\text{(1)} & \quad T-F_f-Mg\sin\theta = Ma \\
\text{(2)} & \quad mg - T = ma \\
\end{align*}
\]
Example of Two-Part Problem

- First: set $F_f = 0$ and solve!

\[
\begin{align*}
1. & \quad T - F_f - Mg \sin \theta = Ma \quad (Mg \cos \theta = N!) \\
2. & \quad mg - T = ma
\end{align*}
\]

\[\Rightarrow \quad -F_f - Mg \sin \theta + mg = (M + m)a\]

\[\Rightarrow \quad a = \frac{(m - M \sin \theta)g - F_f}{M + m} = \frac{(1.5 - 1.0)9.8 - 0}{3.5} = 1.4 \text{ m/s}^2\]

- Thus the motion, in the absence of friction, would be in the positive $x$-direction! (we were “lucky” in our choice)

- Friction will always OPPOSE the incipient motion, so it adds to the tension $T$ in the rope for block $M$.

- With friction:

\[a = \frac{(m - M \sin \theta)g - F_f}{M + m} = \frac{(1.5 - 1.0)9.8 - 3.0}{3.5} = 0.54 \text{ m/s}^2\]

- Note: if we had found a negative value for $a$ here (e.g. with $F_f = 5.0 \text{ N}$), we would again have a problem: kinetic friction only exists by grace of motion, and the $3.0 \text{ N}$ is thus only an upper limit! In that case the system would never even start to move...
Recap Acceleration

• Special case: Constant Acceleration formulae...
  - **Motion:** \( \mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2 \), \( \mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a} t \)
  - **Note:** these are VECTOR equations; i.e. we have equations for EACH COMPONENT!

• Projectile motion: trajectory is a parabola (ignoring drag/friction)
  - The trajectory is found by eliminating \( t \) between the \( x \)- and \( y \)-equations of motion, e.g.:
    
    \[
    x(t) = v_{0x} t \quad \Rightarrow \quad t = x/v_{0x}
    \]
    
    \[
    y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2
    \]
    
    \[
    y(x) = \ldots
    \]

    - It is simple to find range, maximum height, etc., once the INITIAL CONDITIONS \((\mathbf{r}_0, \mathbf{v}_0)\) and the acceleration \(\mathbf{a}\) are known...
Circular Motion

• Motion along a circular path/segment $s$, radius $R$:
  - Motion: $s(t) = s_0 + v_0 t + \frac{1}{2} a_{\parallel} t^2$, $v(t) = v_0 + a_{\parallel} t$
    • true for any (curved/non-curved) path with constant acceleration $a_{\parallel}$
      PARALLEL ($\parallel$) to the path!

- Circular Motion:
  divide the formulae by radius $R$ to get: $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} a t^2$, $\omega(t) = \omega_0 + at$
  with the new DEFINITIONS:
  $\theta(t) \equiv s/R = \text{angular position}$, $\omega(t) \equiv (ds/dt)/R = d\theta/dt = \text{angular velocity}$

  $a \equiv a_{\parallel}/R = \text{angular acceleration}$
  • For Uniform Circular Motion: $a_{\parallel} = 0$

- Because (EVEN if $a_{\parallel}$ is constant!) the velocity's DIRECTION is changing, there MUST be a non-zero acceleration!
  • centripetal acceleration ($\perp$ to path): $a_{\perp} = v^2/R$ (derivation next slide)

- TOTAL acceleration: $\mathbf{a} = a_{\parallel}\mathbf{i} + a_{\perp}\mathbf{j}$
  (addition of perpendicular vectors!)
**Centripetal Acceleration**

- **Is the component of the total acceleration directed towards the center of the circular motion...**

- **Consider UNIFORM circular motion, i.e. \( a_{//} = 0 \), and thus** \( v(t) = v(t + \Delta t) \):
  - in a time \( \Delta t \), the object moves along the circle over a distance \( \Delta s : v = \Delta s / \Delta t \).
  - in that time the angular position vector rotates by angle \( \Delta \theta \)
  - Note that (in RADIANS!) \( \Delta \theta = \Delta s / R = \Delta v / v \)
  - The DIRECTION of \( \Delta v = a \Delta t \) is towards the CENTER of rotation
  - Combining:
    \[
    a = a_{\perp} = \Delta v / \Delta t = v \Delta \theta / \Delta t = v \Delta s / \Delta t / R = v^2 / R = \omega^2 R
    \]
Example…

- A car is going through a turn of radius $R=40$ m (about 130 ft). At a certain point in the turn the car has a speed of 36 mi/hr (i.e. $v=36 \times 1650 \text{m} / \text{3600 s} = 16.5 \text{ m/s}$), and is accelerating with 1.0 m/s$^2$ in the direction of motion: $a_{//} = 1.0 \text{ m/s}^2$.

  - Q1: calculate the centripetal acceleration $a_{\perp}$ at that time

$$a_{\perp} = \frac{v^2}{R} = \frac{(16.5 \text{ m/s})^2}{40 \text{ m}} = 6.8 \text{ m/s}^2$$

  - Note that this is large, about 70% of $g$!

  - Q2: calculate the TOTAL acceleration.

    - Note: acceleration is a VECTOR, and thus I must specify either the components (in a well-defined axis system) or the magnitude and direction (with respect to a pre-defined axis)

$$\mathbf{a} = a_{//} \mathbf{i}_{//} + a_{\perp} \mathbf{j}_{\perp} = (1.0 \mathbf{i}_{//} + 6.8 \mathbf{j}_{\perp}) \text{m/s}^2,$$

or:

$$a = \sqrt{a_{//}^2 + a_{\perp}^2} = \sqrt{1.0^2 + 6.8^2} = 6.9 \text{ m/s}^2 \text{ (magnitude)}$$

$$\varphi = \arctan \left( \frac{a_{\perp}}{a_{//}} \right) = \arctan \left( \frac{6.8}{1.0} \right) = 82^\circ \text{ w.r.t motion (direction)}$$

(inwards)
Example

• I’m sitting in a vertical Ferris Wheel of radius R. At the top of the ride I feel “weightless”. Calculate the speed of the wheel...

• Sketch:

• Discuss:
  - “Weightless” means I don’t feel the seat pressing up onto my pants: $N=0$!
  
  - NET FORCE is therefore $W$!

  - Thus: $W=mg=ma=mv_0^2/R \Rightarrow a=v_0^2/R=g \Rightarrow v_0=\sqrt{gR}$!
Classical Physics I

PHY131
Lecture 9
Circular motion: dynamics
Forces in Circular Motion

- We saw before:
  \[ a_{\text{rad}} = a_c = \frac{v^2}{R} \]
  This was derived from:

- Direction of \( a \): pointing towards center of rotation! (centripetal)
- If there is an \( a_c \), there must be a Net Force \( F_c \) causing it! (2\textsuperscript{nd} Law)
Free Body Diagram

- net force for \textit{uniform} circular motion with speed $v$ on a circle of radius $R$:

$$F_{\text{Net}} = ma_{\text{rad}} = ma_c = m \frac{v^2}{R}$$

(directed to the center of the circle!)

- a simple way of “deriving” this is by use of units:

$[a] = \text{m/s}^2 \Rightarrow \text{using } v \text{ and } R \Rightarrow a \propto \frac{[v^2]}{[R]} = \frac{\text{m}^2/\text{s}^2/\text{m}}{\text{m}} = \text{m/s}^2$

- in case of \textit{non-uniform} circular motion, with instantaneous speed $\nu$:

$$F_{\text{Net}} = ma = m(a_c + a_{//})$$

with: $a_c = \frac{v^2}{R}$
Example 1

- A car is going through a turn of radius $R=40$ m (about 130 ft). At a certain point in the turn the car has a speed of 36 mi/hr (i.e. $v=36*1650\text{m}/3600\text{s}=16.5\text{m/s}$), and is accelerating with $1.0\text{m/s}^2$ in the direction of motion: $a_{//}=1.0\text{m/s}^2$.

  - Q1: calculate the centripetal acceleration $a_{\perp}$ at that time
    \[
    a_c = \frac{v^2}{R} = \frac{(16.5\text{m/s})^2}{40\text{m}} = 6.8\text{m/s}^2
    \]
    - Note that this is large, about 70% of $g$!

  - Q2: calculate the TOTAL acceleration.
    - Note: acceleration is a VECTOR, and thus I must specify either the components (in a well-defined axis system) or the magnitude and direction (with respect to a pre-defined axis)
      \[
      \vec{a} = a_{//}\hat{i}_{//} + a_c\hat{j}_\perp = (1.0\hat{i}_{//} + 6.8\hat{j}_\perp)\text{m/s}^2, \quad \text{or:}
      \]
      \[
      a = \sqrt{a_{//}^2 + a_c^2} = \sqrt{1.0^2 + 6.8^2} = 6.9\text{m/s}^2 \text{ (magnitude)}
      \]
      \[
      \varphi = \arctan\left(\frac{a_c}{a_{//}}\right) = \arctan\left(\frac{6.8}{1.0}\right) = 82^\circ \text{ w.r.t motion (direction)} \quad \text{(inwards)}
      \]
Example 2

• I’m sitting in a vertical Ferris Wheel of radius R. At the top of the ride I feel “weightless”. Calculate the speed of the wheel...

• Sketch:

• Discuss:
  - “Weightless” means I don’t feel the seat pressing up onto my pants: $N=0$!
  - NET FORCE is therefore $W$!
  - Thus: $W=mg=ma=mv_0^2/R \Rightarrow a=v_0^2/R=g \Rightarrow v_0=\sqrt{(gR)}$!
Example: hockey puck on a string

- Consider a hockey puck on a sling without friction on a horizontal sheet of ice. The puck is tied with a massless string to a central post, and is sliding with a constant speed $v$ along a circular path of radius $R$.

  - Free body diagram:

  - $a_y = 0$
    - $W + N = 0$
    - $N = mg$

  - $\mathbf{F}_{Net} = F_c = ma_c = mv^2/R \hat{i}$
    where $v$ is the (constant) speed of the puck in its circular trajectory of radius $R$

  - From this we can build all kinds of problems...
Conical Pendulum – Next week’s Lab

• see book, study yourself...
• quite similar to the next example: car on banked curve...
For a car of mass $m$ going through a (circular) curve in the road, a net force is required to keep them on a circular trajectory.

- This centripetal (=NET) force consists of two forces:
  - the horizontal component of the friction force of the road on the car, plus
  - the horizontal inward-pointing component of the normal force $N \sin \theta$ if the curve is banked...

- Note, that the Normal force is a reaction force

- Until the car starts slipping sideways, the friction force is the force of static friction between the tires and the road: $F_f = \mu_s N$
Car on a Banked Curve

- Free-Body diagram:

- Q: what is the ideal speed for this banked curve? (i.e. when friction is not needed...)

\[ F_{\text{net}} = ma \] is 2 equations:

**Vertical:**

\[ ma_y = 0 = N \cos \theta - W \mp F_f \sin \theta = N \cos \theta \mp \mu_s N \sin \theta - mg \]

\[ \Rightarrow N = \frac{mg}{\cos \theta \mp \mu_s \sin \theta} \]

**Horizontal:**

\[ ma_x = ma_{\text{rad}} = m \frac{v^2}{R} = F_{\text{Net},x} = N \sin \theta \pm F_f \cos \theta = N \left( \sin \theta \pm \mu_s \cos \theta \right) \]

\[ \Rightarrow v^2 = gR \frac{\sin \theta \pm \mu_s \cos \theta}{\cos \theta \mp \mu_s \sin \theta} = gR \frac{\tan \theta \mp \mu_s}{1 \mp \mu_s \tan \theta} \]

- Thus, for \( \mu_s = 0 \), \( v = \sqrt{(gR \tan \theta)} \); check!
- For \( \theta = 0 \), we **need** friction: \( v_{\text{max}} = \sqrt{(\mu_s gR)} \); check!
- Else: \( v_{\text{min}} < v < v_{\text{max}} \) with:

\[ v^2_{\text{min}} = gR \frac{\tan \theta - \mu_s}{1 + \mu_s \tan \theta}, \quad v^2_{\text{max}} = gR \frac{\tan \theta + \mu_s}{1 - \mu_s \tan \theta} \]

- e.g: for \( \mu_s = 0.70 \), \( \theta = 10^\circ \), \( R = 30 \text{ m} \) we find:

\( v_{\text{max}} = 17 \text{ m/s (37 mi/hr)} \) and \( v_{\text{min}} = 0 \text{ m/s} \)

Note: the normal force is a REACTION force!
Example

- A block of mass slides along a horizontal surface lubricated with a thick oil which provides a drag force proportional to the square root of velocity: \( F = bv^{1/2} \)

- **Q:** It starts with \( v_0 \); how far does the block slide?

\[
F_{\text{net}} = -b\sqrt{v} = ma = m \frac{dv}{dt} \quad \Rightarrow \quad -\frac{b}{m} \int_0^t dt = \int_0^{v_0} \frac{dv}{\sqrt{v}}
\]

\[
\Rightarrow \quad -\frac{b}{m} t = \left[ 2\sqrt{v} \right]_0^{v_0} = -2\sqrt{v_0}
\]

\[
F_{\text{net}} = -b\sqrt{v} = ma = m \frac{dv}{dt} = mv \frac{dv}{dx} \quad \Rightarrow \quad -\frac{b}{m} \int_0^D dx = \int_0^{v_0} \sqrt{v} dv
\]

\[
\Rightarrow \quad -\frac{b}{m} D = \left[ \frac{2}{3} \frac{v^3}{2} \right]_0^{v_0} = -\frac{2}{3} v_0^{3/2}
\]