
a. The direction of the torque due to $F$ around $A$ is …

b. When a spring has positive potential energy, it must be …

c. The moment of inertia of a thin uniform stick of mass $M$ and length $L$ for a rotation axis $\perp$ the stick through its end equals …

d. If the potential energy function $U_F(r)$ of a force is known, the force $F(r)$ itself is …

e. With the usual choice of $U_G=0$ at infinity, $U_G$ of a satellite in an orbit around Earth is …

f. The center of mass of the flat uniform figure is located at …

g. A freely rotating object always rotates around …

2. Work (25 points). A block of mass $m=2.0$ kg slides on a flat horizontal surface, with unknown coefficient of kinetic friction $\mu_k$, towards a horizontally mounted ideal spring with spring constant $k=48$ N/m. The initial speed of the block is $v_0=6.0$ m/s and the initial distance between block and spring is $D=5.5$ m. The maximum spring compression after the block’s collision is $d=0.50$ m. For ease of calculus, take $g=10$ m/s$^2$.

a. Here, is the work done by friction positive or negative?

b. The magnitude of the work done by friction, from start till maximum spring compression, equals:

c. The initial energy of the block equals:

d. The coefficient of kinetic friction is:
3. **Impulse (15 points).** A uniform thin stick of known mass $M$ and known length $L$ is resting on ice (assumed to be perfectly frictionless). I give it a sharp horizontal kick (i.e. impulse) of known magnitude $J$, perpendicular to the stick at one of its ends …

   a. Calculate the speed of the stick’s center-of-mass after the kick in terms of the known quantities:

   - $2J/(ML)$ (2 p)
   - $J/M$ (7 p)
   - $JML/2$ (2 p)
   - $JL/2$ (2 p)
   - $JM$ (2 p)

   b. Calculate the number of rotations per second of the stick after the kick:

   - $3J/(4\pi ML)$ (3 p)
   - $3J/(2\pi ML)$ (3 p)
   - $3J/(\pi ML)$ (8 p)
   - $2J/(\pi ML)$ (2 p)
   - $6J/(\pi ML)$ (3 p)

4. **Gravity (25 points).** A 500 kg satellite is to be placed in a circular orbit with radius $7R_E$ around Earth ($M_E = 5.98 \times 10^{24}$ kg, $RE = 6.4 \times 10^6$ m). The launch platform is at the equator and at sea level $r=RE$.

   a. Calculate the kinetic energy of the satellite before launch:

   - $+2.226 \times 10^9$ J
   - $+5.415 \times 10^7$ J (6 p)
   - $-4.452 \times 10^9$ J
   - $-3.111 \times 10^{10}$ J
   - $-3.116 \times 10^{10}$ J

   b. Calculate the potential energy of gravity of the satellite before launch:

   - $+2.226 \times 10^9$ J
   - $+5.415 \times 10^7$ J
   - $-4.452 \times 10^9$ J
   - $-3.111 \times 10^{10}$ J (3 p)
   - $-3.116 \times 10^{10}$ J (6 p)

   c. Calculate the potential energy of gravity of the satellite in its final orbit:

   - $+2.226 \times 10^9$ J
   - $+5.415 \times 10^7$ J
   - $-4.452 \times 10^9$ J (6 p)
   - $-3.111 \times 10^{10}$ J
   - $-3.116 \times 10^{10}$ J

   d. Calculate the kinetic energy of the satellite in its final orbit:

   - $+2.226 \times 10^9$ J (7 p)
   - $+5.415 \times 10^7$ J
   - $-4.452 \times 10^9$ J
   - $-3.111 \times 10^{10}$ J
   - $-3.116 \times 10^{10}$ J
Forces

Momentum

Perpendicular-Axis Theorem for a Parallel-Axis Theorem

Power & Collisions:

Circular motion (radius \( r \), angular velocity \( \omega \), rotation angle \( \theta \)) with constant \( \alpha \):

\[ \omega = \omega_0 + \alpha t; \quad \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2; \]

eliminating \( t \): \[ \omega^2 = \omega_0^2 + 2\alpha (\theta - \theta_0) \]

Circular motion – radial acceleration \( a_{rad} \):

\[ a_{rad} = a_c = v_T^2/R \text{ (radially inwards)} \]

Center-of-Mass position of system (mass \( M \))

\[ \mathbf{r}_{cm} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i} = \frac{1}{M} \int dm \mathbf{r}, \quad \mathbf{v}_{cm} = \frac{1}{M} \int \mathbf{v} dm \]

Moment of Inertia \( I \):

\[ I = \frac{\sum m_i r_i^2}{\sum m_i} = \frac{1}{M} \int r^2 dm \]

Parallel-Axis Theorem

\[ I = I_{cm} + M d^2 \]

for a Planar (-flat) body in the \( x-y \) plane: \( I = I_x + I_y \)

Momentum \( \mathbf{p} \)

\[ \mathbf{p} = \sum m_i \mathbf{v}_i = M \mathbf{v}_{cm} \]

Angular Momentum \( \mathbf{L} \)

\[ \mathbf{L} = \sum m_i \mathbf{r}_i \times \mathbf{v}_i = I \omega \text{ for rotation around symmetry axis} \]

Kinetic Energy \( K \) [J≡Nm]:

Moving axis: \( K_{tot} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega_{cm}^2 \)

Fixed axis \( A \): \( K_{rot,A} = \frac{1}{2} I_A \omega_A^2 \)

Forces [N≡kgm/s²] and consequences:

\[ \sum f_i = \frac{d \mathbf{p}}{dt} = m \mathbf{a}; \quad \mathbf{F}_{on,B} = -\mathbf{F}_{on,A}; \quad \sum \tau_i = L/dt = I \alpha \]

Force of Gravity between \( M \) and \( m \), at center-to-center distance \( r \):

\[ F_G = GMm/r^2 (\text{r}) \text{ (attractive! } G=6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2) \]

near sea level: \( F_G = mg(\text{downwards; } g=9.80 \text{ m/s}^2) \)

Force of a Spring (spring constant \( k \)):

\[ F_s = -kx \]

(opposes compression/stretch \( x \))

Friction: static: \( F_f \leq \mu_s N \), kinetic: \( F_f = \mu_k N \), opposes motion; \( \mu = \text{frict'}n \text{ coef.} \); \( N = \text{normal force} \)

Torque:

\[ \tau = \mathbf{R} \times \mathbf{F} \]

\( \tau = RF \sin \theta_{RF} \text{ (right-hand rule)} \)

Equilibrium & Collisions: Elastic: \( K \) is conserved; Completely Inelastic: objects stick together afterwards

Impulse by a force \( \mathbf{F} \) over a time interval:

\[ \mathbf{J}_F \equiv \int \mathbf{F} dt \]

Work done by a force \( \mathbf{F} \) over a trajectory:

\[ W_F \equiv \int \mathbf{F} \cdot \mathbf{dx} \]

Work done by a torque \( \tau_F \) over a rotation angle \( \theta \):

\[ W_F \equiv \int \tau_F \cdot d\theta \]

Work-Kinetic Energy relationship (from \( \sum \mathbf{F} = m \mathbf{a} \)):

\[ W_{tot} = \sum W_i = \Delta K = K_f - K_i \]

Power \( P \) [W≡J/s]:

\[ P_F \equiv dW_F/dt = \mathbf{F} \cdot \mathbf{v} = \tau_F \cdot \omega \]

Potential Energy \( U \) of a conservative force \( \mathbf{F} \):

\[ U_F = -W_F; \quad \text{e.g. } U_G = -GMm/r, \quad U_S = \frac{1}{2} kx^2 \]

Work by Non-Conservative forces - Total Energy:

\[ W_{NC} = \Delta E = E_f - E_i \equiv K_f + \sum U_i - (K_i + \sum U_i) \]