Observation: the pressure $p$ in fixed-volume $V$ filled with a 'thin' gas varies linearly with temperature.

This is an example of the "**IDEAL GAS LAW**" (which we’ll discuss later):

$$pV = nR(T+T_0),$$

- where $R = 8.314 \text{ J/}^\circ\text{C/mole}$ is the **GAS CONSTANT**,  
- and $n$ is the number of moles of gas;  
- the mass of the gas $m = nM$, where $M$ is the **MOLAR MASS**

$$pV=nRT=(m/M)RT$$  
([T]=\text{K})

- For ideal gases the extrapolation always goes to the same point at $p = 0 \text{ atm}$
- The temperature where the pressure extrapolates to zero is the **ABSOLUTE ZERO** temperature $T_0 = -273.15 ^\circ\text{C}$, and is the zero point of the **KELVIN [K]** temperature scale:  
  $$T_K = T_C + 273.15 \text{ (size of K and } ^\circ\text{C are equal!!)}$$
- We cannot actually reach ABSOLUTE ZERO, but can get very close: e.g. He liquifies only at 4.3 K. Temperatures of $\mu$K's have been reached …
Heat

Heat, symbol $Q$, is energy transferred
INTO ($Q>0$), or OUT OF ($Q<0$) a system

- SI units: $[J] = [Nm]$
- Other (historical) units:
  - 1 cal(-orie) = 4.186 J (defined as the energy needed to heat 1 g = 1 cc of water by 1 °C)
  - 1 food calorie = 1 kcal = 4186 J
  - 1 BTU (British Thermal Unit) = 1055 J

- when discussing SMALL heat transfers, e.g. giving rise to small temperature changes $dT$, we use notation: $dQ$
Heat Flow and Temperature change

Heat results in either a ...

- **temperature change** of matter,

ór a ...

- **change of the “phase” of matter at constant temperature**; i.e.:
  - vaporization - condensation: liquid $\leftrightarrow$ gas,
  - liquefaction (melting) - freezing: solid $\leftrightarrow$ liquid,
  - sublimation: solid $\leftrightarrow$ gas

Temperature change due to Heat; Empirically:

- $dQ = mc \, dT$ (when no phase transformation occurs), where $dQ$ is the (small) heat transfer, $dT$ the resulting (small) temperature change, $m$ the mass [kg] of the material, and $c$ a material-specific proportionality constant.
  - $c$ is called “**SPECIFIC HEAT**” of the material, and is often constant over a good temperature range, but sometimes depends on $T$ itself.
  - e.g. for **water**: $c = 1 \text{ cal/g/°C} = 4186 \text{ J/kg/°C} = 4186 \text{ J/kg/K} = 1 \text{ BTU/lb/°F}$
  - for **ice**: $c = 2100 \text{ J/kg/°C}$
  - For most materials: $c = 100-5000 \text{ J/kg/K}$
Heat and Temperature

Phase Change due to Heat: Empirically:
\[ Q = mL \]
when a phase transformation occurs, where \( Q \) is the heat transfer, \( m \) the mass [kg] of the material, and \( L \) a material-specific proportionality constant.

- \( L \) is the “LATENT HEAT” of VAPORIZATION (\( L_v \)), or FUSION (\( L_f \))
- for water (at 1 atm):
  \( L_v = 2.256 \times 10^6 \) J/kg (water ⇋ vapor); \( L_f = 0.334 \times 10^6 \) J/kg (water ⇋ ice)
- For most materials: \( L = 100-5000 \) kJ/kg

Example: how much heat is required to vaporize 0.50 kg of ICE at −10 °C? How long does it take with a 500 W heater?

- Ice −10 °C → 0 °C:
  \[ Q = mc_{\text{Ice}} \Delta T = 0.50 \text{ kg} \times 2100 \text{ J/kg/K} \times 10 \text{ K} = 10.5 \text{ kJ} \]
- Ice 0 °C → Water 0 °C (Phase change):
  \[ Q = mL_{f,\text{Ice}} = 0.50 \text{ kg} \times 334 \text{ kJ/kg} = 167 \text{ kJ} \]
- Water 0 °C → 100 °C:
  \[ Q = mc_{\text{Water}} \Delta T = 0.50 \text{ kg} \times 4186 \text{ J/kg/K} \times 100 \text{ K} = 209 \text{ kJ} \]
- Water 100 °C → Vapor 100 °C (Phase change):
  \[ Q = mL_{v,\text{Water}} = 0.50 \text{ kg} \times 2256 \text{ kJ/kg} = 1128 \text{ kJ} \]
- Total: \( Q = 1515 \text{ kJ} \)
- With a \( P = 500 \text{ W} = 500 \text{ J/s} \) heater:
  \[ P = \frac{\Delta E}{\Delta t} = \frac{Q}{\Delta t} \Rightarrow \Delta t = \frac{Q}{P} = 1515 \text{ kJ} / 0.500 \text{ kJ/s} = 3030 \text{ s} = 51 \text{ min} \]
Example

80 g of an unknown material \( X \) at \( T = 100 \, ^\circ\text{C} \) is dunked in a 150 g Aluminum cup containing 200 g of water at 19 \( ^\circ\text{C} \). After equilibrium is established, all three materials are at 26 \( ^\circ\text{C} \).

Assuming no heat is lost to the surroundings, calculate the specific heat of the unknown material.

- Energy is conserved (no heat loss to surroundings), thus the sum of all heat (flows) is zero!
- In equation form: \( Q_x + Q_{\text{water}} + Q_{\text{cup}} = 0 \)

- Working this through:
  \[
  m_x c_x (26 - 100) \, ^\circ\text{C} + (m_w c_w + m_c c_c) (26 - 19) \, ^\circ\text{C} = 0
  \]
  \[
  0.080 \times c_x (26-100) + (837.2+0.150\times390)(26-19) = 0
  \]
  \[
  c_x = (837.2+0.150\times390)7/(0.080\times74) = 1059 \, \text{J/kg/K}
  \]
Heat Flow

Heat flow $H \equiv \frac{dQ}{dt}$ [W] can occur by three different means, often occurring simultaneously:

**Conduction** is flow of energy through materials (e.g. transported by quasi-free electrons in metals, and by lattice vibration waves (phonons):

$$H = \frac{dQ}{dt} = k A \left( T_H - T_L \right)/L,$$

- with $A$ the cross sectional area of the heat conductor (perpendicular to the heat flow),
- and $L$ the total length of the heat conductor (parallel to the heat flow),
- and $T_H$ ($T_L$) the high and low temperatures at the two sides of the material,
- and the proportionality constant $k$ is the material-dependent **heat conductivity** of the material; SI units: [W/K/m]
- Also used: the **heat resistance** $R \equiv 1/k$ (“R”-value, USA units [ft² °F hr/BTU])

**Convection** is the primary mechanism of heat transport in fluids and gases: Approximately:

$$H \propto A \left( T_H - T_L \right)^{5/4}$$
Radiation is energy flow by emission or absorption of electromagnetic radiation (light, infra-red radiation, ...):

\[ H = \frac{dQ}{dt} = eA\sigma T^4, \]
- with \( A \) the total area of the body emitting or absorbing the radiation,
- and \( T \) the KELVIN temperature of the emitting body,
- the proportionality constant is \( \sigma = 5.6705 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4} \), the universal Stefan-Boltzmann constant,
- and \( e \) is the material-dependent emissivity or absorption coefficient \((0 < e < 1)\). For rough and black materials \( e \approx 1 \)

Note that EVERY object at \( T_K > 0 \) emits radiation; including the surroundings; the NET heat flow into a body is thus:

\[ H = \frac{dQ}{dt} = eA\sigma(T_s^4 - T_b^4), \]
- with \( T_b \) is the Kelvin temperature of the body, and \( T_s \) the Kelvin temperature of the surroundings!
- Typically, radiation loss is small compared to other losses ...
One end of a 20 cm long uniform copper bar is kept at 20.00 K, while the other end is blackened and exposed to surroundings at 500 K. Assuming only radiative flow occurs between the surroundings and the blackened end of the bar, calculate the temperature of the blackened end.

(Hint: the temperature will be close to 20 K because copper is a good heat conductor)

- The net radiative heat flow into the blackened end (emissivity $e=1$) of the bar equals the heat conduction through the bar:

$$H_{\text{rad}} = H_{\text{cond}}$$

$$\sigma A (500^4 - T^4) = k A (T - 20.00)/L$$

$$5.6705 \times 10^{-8} (500^4 - T^4) = (1670/0.20) \Delta T$$

for $T \approx 20$ K, $T^4 << 500^4$ and can be ignored on the left-hand side!

Thus:

$$\Delta T = 5.6705 \times 10^{-8} (500^4) / (1670/0.20) = 0.42$$

$\Rightarrow T = 20.42$ K
Equation of State

First: remember mass and density:
- mass: \( m = n \cdot M \) [kg], with \( n \) the number of moles [mol] and \( M \) the molar mass [kg/mol]
- density: \( \rho \equiv \frac{m}{V} \) [kg/m³]
- 1 mol of a stuff is defined as \( N_A = 6.022 \times 10^{23} \) molecules of it (or as the number of atoms in exactly 12 grams of Carbon-12)

Equation of State for an “IDEAL GAS”: \( pV = nRT \)
- where \( p \) is pressure, \( V \) is volume, \( T \) is the \textit{Kelvin} temperature, \( n \) is # moles, and \( R = 8.3145 \) J/mol/K is the “\textit{GAS CONSTANT}”
- for 1 mol of an (ideal) gas at \textit{STP} (Standard Temperature \( T = 0 \) °C, and Pressure \( p = 1 \) atm = \( 1.013 \times 10^5 \) Pa):
  \[ V = 1 \times 8.3145 \times 273.15 / 1.013 \times 10^5 = 22.41 \text{ L} \]
- \( \Rightarrow pV/T \) is constant for fixed amount of gas!

for not-so-ideal gas (van der Waals): \( [p - a(n/V)^2]V[1 - b(n/V)] = nRT \)
- e.g. \textit{CO}_2: \( a = 0.364 \) Nm⁴/mol², \( b = 4.27 \times 10^{-5} \) m³/mol
- Average distance between molecules at \textit{STP}:
  \[ \frac{V}{N_A} = 22.4 \times 10^{-3} / 6.022 \times 10^{23} = 3.7 \times 10^{-26} \text{ m}^3 = L^3 \Rightarrow L = 3.3 \text{ nm (} \sim 30-60 \times \text{ atomic dimension)} \]
State and Phase Diagrams

Substances exist in three phases:
- solid,
- liquid,
- gas/vapor

Changing volume $V$, pressure $p$, and or temperature $T$ allows us to “scan” the whole phase space, which is a surface in 3-DIMENSIONAL space!
Example

Given the density of outside air (at STP) $\rho_{\text{cold}}$, calculate the required temperature of the hot air $T_{\text{hot}}$ in a hot-air balloon of volume $V$, and mass $m_B$ (balloon skin, burners, payload) for take-off...

- to take off, the weight of hot air, payload, etc must be cancelled by the Buoyant force from the surrounding cold air ...

- Equilibrium ($m_{\text{hot}}$ is the mass of the hot air):

  $$ m_{\text{hot}}g + m_B g = \rho_{\text{cold}} V g $$
  $$ \Rightarrow m_{\text{hot}} = \rho_{\text{cold}} V - m_B \Rightarrow m_{\text{hot}}/V = \rho_{\text{hot}} = \rho_{\text{cold}} - m_B/V $$

- treating the air as an ideal gas, the number of moles $n$ of hot air in the balloon as function of temperature is:

  $$ pV = nRT = (m/M)RT $$
  $$ \Rightarrow T_{\text{cold}} = p(V/m)(M/R) = (pM/R)/\rho; $$
  $$ \Rightarrow T_{\text{hot}}/T_{\text{cold}} = \rho_{\text{cold}}/\rho_{\text{hot}}; \quad T_{\text{hot}} = T_{\text{cold}}/(1 - m_B/\rho_{\text{cold}} V) $$