Electricity and Magnetism

Maxwell’s Electromagnetic Waves:

• Important consequence: one can calculate the velocity of electromagnetic waves based on properties of capacitors and inductors!

\[ v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \equiv c \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Event/Source</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1675</td>
<td>Rømer and Huygens</td>
<td>moons of Jupiter 220000</td>
</tr>
<tr>
<td>1729</td>
<td>James Bradley</td>
<td>aberration of light 301000</td>
</tr>
<tr>
<td>1849</td>
<td>Hippolyte Fizeau</td>
<td>toothed wheel 315000</td>
</tr>
<tr>
<td>1862</td>
<td>Léon Foucault</td>
<td>rotating mirror 298000±500</td>
</tr>
<tr>
<td>1907</td>
<td>Rosa and Dorsey</td>
<td>EM constants 299710±30</td>
</tr>
<tr>
<td>1926</td>
<td>Albert A. Michelson</td>
<td>rotating mirror 299796±4</td>
</tr>
<tr>
<td>1950</td>
<td>Essen and Gordon-Smith</td>
<td>cavity resonator 299792.5±3.0</td>
</tr>
<tr>
<td>1958</td>
<td>K.D. Froome</td>
<td>radio interferometry 299792.50±0.10</td>
</tr>
<tr>
<td>1972</td>
<td>Evenson et al.</td>
<td>laser interferometry 299792.4562±0.0011</td>
</tr>
<tr>
<td>1983</td>
<td>17th CGPM</td>
<td>definition of the meter 299792.458 (exact)</td>
</tr>
</tbody>
</table>

In Maxwell’s words:

“This velocity is so nearly that of light, that it seems we have strong reasons to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws.”

Ref: Wikipedia.com
From Maxwell to Einstein

• Finished PhD 1901.
• Working in Swiss Patent Office while completing “Habilitation” so that he could seek a Professorship at a German university.
• Then: the revolution of 1905, what a year!
  - Abraham Pais' book “Subtle is the Lord”

Albert Einstein
(1879-1955).
Nobel Prize: 1921
Einstein’s Revolution #1

Takes Maxwell at his word: speed of light in vacuum $c = 1/\sqrt{\mu_0 \varepsilon_0}$

This is *independent* of any frame of reference!

cfr. shining a beam forward or backwards in a moving car ... !

**Consequences:**

- time and distance measurements appear different in differently moving frames
  - **Lorentz contraction:**
    - Rulers $L_0$ appear to *shrink* when moving at fast speed $v$: $L' = L_0 / \gamma$, with
      \[
      \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \approx 1 + \frac{1}{2} \frac{v^2}{c^2} \geq 1
      \]
  - **Time Dilation:**
    - Clocks appear to *slow down* when moving at fast speed $v$: $t' = \gamma t_0$

- **Energy-mass equivalence:**
  \[
  E^2 = (pc)^2 + (m_0 c^2)^2 \Rightarrow \gamma = E / (m_0 c^2)
  \]

- Now known as **Special Relativity**
  - *(General Relativity deals with large acceleration between frames of reference, while Special Relativity deals with large velocity between frames of reference)*
Relativistic Effects are Everyday!

Cosmic ray muons are produced at a height of \( \sim 10-20 \text{ km} \) in the atmosphere, in primary interactions of cosmic rays (energetic protons, nuclei, and X-rays):

- At rest, they live for 2.2 \( \mu \text{s} \) on average
- without relativity, they would travel on average \( (v \approx c = 300 \text{ m/\mu s}) \) 660 m < 1 km, and NEVER reach sea-level

**Observation:** MANY muons reach sea-level!

- muons are a main component of cosmic radiation at sea-level!
- only explained by:
  - TIME-DILATION (of their life-clocks)
  - LENGTH-CONTRACTION (of the atmospheric depth seen by the speeding muons)
England and fine china: how hot is a red-hot or white-hot oven?

Question: Can one predict the oven’s radiation spectrum from temperature, or vice versa?

• First attempts: predict a blow-up on the short-wavelength end.

• Refined attempts: much closer but still not quite right.

• Was a major puzzle in physics at the end on the 19th century
**That’s odd...**

- **Wien, 1896:** semi-empirical fit that was pretty close
- **Max Planck:** ties some aspects of Wien’s law to statistical thermodynamics.
- **But not quite right!** Heinrich Rubens of Universität Berlin found discrepancies with his own measurements.
- **Oct. 7, 1900:** Rubens pays a Sunday afternoon visit to Planck. By that evening, Planck has found an answer that works.

---

Sample of the Rubens–Kurlbaum data which led Planck to guess his radiation formula [R1]. $\rho$ is plotted versus $T$ for $\lambda = 51.2\mu$m. (“berechnet nach” means “computed after”, “beobachtet” means “observed”). The curves marked “Wien” and “Lord Rayleigh” refer to best fits to the Eqs. (19.5), (19.17), respectively. The curves marked “Thiesen” and “Lummer-Jahnke” refer to theoretical proposals which are not discussed in this book. Planck’s formula is not yet plotted.
Planck’s theory

Imagine a black body made of a conductor (metal): Electromagnetic standing waves must have nodes at boundaries.

- Associate the Energy of each wave with its Frequency: \( E = hf \).
  - Planck’s constant: \( h = 6.63 \times 10^{-34} \) Js
  
→ Energy and Wavelength: \( E = h \frac{c}{\lambda} \)

\[ hc = 1240 \text{ eV} \cdot \text{nm} \]

- Add all possible waves and their energies: with well-established statistical thermodynamics, this gives a perfect fit!

- But quite a radical solution! Energy is quantized!

Max Planck (1858-1947). Nobel Prize: 1918

and wavelength must be matched to cavity dimensions!
The Difference is Subtle…

In fact, some people don’t even notice!

• Lord Rayleigh (John William Strutt) proposes corrections to Wien’s law in 1900
• James Hopwood Jeans contributes further in 1905!
• Will anyone take Planck seriously?
The Temperature of the Universe

... is 2.725 K

- VERY close to Black Body spectrum and VERY uniform across the sky but with statistically significant tiny fluctuations on specific length scales

- which fits perfectly with a BIG BANG scenario!
Planck’s Revolution was that …

1. Light consists of electromagnetic waves
2. Light comes as photons of energy $E=hf$
3. The black-body radiation curve goes to zero at small wavelength
4. Monochromatic light exhibits interference
5. The peak of the black-body radiation spectrum moves with temperature
The Photoelectric Effect …

Observation: light impinging on some metals (in vacuum) can eject electrons
- useful in measuring light levels - budding photographic industry!
- experimental set-up:

- Observations:
  - there is a THRESHOLD frequency; below $f_0$ there is NO ELECTRON EMISSION
  - The INTENSITY of the light does NOT affect the threshold
  - ABOVE the threshold: current is proportional to the light’s intensity

Threshold behavior with frequency, not intensity, was a real puzzle
**Einstein’s Revolution #2**

Einstein takes Planck at his word: \( E = hf \)

- Special relativity for light (no mass) then gives
  \[
  E = hf = \sqrt{(mc^2)^2 + (pc)^2} = pc
  \]

- And thus photons carry momentum:
  \[
  p = \frac{h}{\lambda}
  \]

This explains the Photoelectric Effect:

- light must be above a threshold frequency, i.e. above a THRESHOLD ENERGY, to knock electrons off a metal surface in vacuum

- Einstein’s 1921 Nobel Prize!
In the photoelectric effect, why does red light not cause the emission of an electron though blue light can?

A. Red photons don’t have sufficient energy to eject an electron.
B. The electric field of the red light oscillates too slowly to eject an electron.
C. Red light contains fewer photons than blue light, not enough to eject electrons.
D. Red light doesn’t penetrate far enough into the metal electrode.
A photon of large enough energy (greater than the work function of the metal) may eject an electron …

- **Photon energy:**
  \[ E = hf \]

- an electron bound in the metal may absorb the photon …

- and be ejected IF:
  \[ E \geq E_0 = hf_0 \]

- the *excess* photon energy goes towards KINETIC energy \( K \) of the electron:
  \[ K = E - E_0 = h(f - f_0) \]

\( K \) can be measured by reversing the potential: “stopping” potential
Monochromatic light shines on the cathode in a photoelectric effect setup, causing the emission of electrons. If the **frequency** of the light stays the same but the **intensity** of the light shining on the cathode is increased ...

A. there will be more electrons emitted.
B. the electrons will be emitted with a higher speed.
C. both A and B are true.
D. neither A nor B are true.
X-ray diffraction is one of the most important tools in chemistry and material science:

the study of the x-ray diffraction pattern from a crystal illuminated by a (monochromatic) x-ray beam allows us to deduce the inter-atomic and inter-molecular distances, and hence find the details of the molecular structure...

- An x-ray beam on a crystal plane (acts as a diffraction grating):
  
  - constructive interference from adjacent lattice sites occurs for \( \theta_{\text{inc}} = \theta_{\text{refl}} = \theta \)

  - x-rays are penetrating; constructive interference from underlying crystal “planes” when \( 2d \sin \theta = m \lambda \),

  - giving x-ray diffraction spots!

- Note1: phase changes occur for all rays equally and are thus NOT an issue!

- Note2: in x-ray diffraction, the angles are conventionally defined with respect to the plane, not the normal!

- Note3: many different “planes” exist \( \Rightarrow \) many spots!
Diffraction = Interference from a Single Slit

Diffraction = Interference between waves coming from different regions in the same slit!

- subdivide the slit of size \( a \) into, for instance, 12 smaller “slits”.

Wavelets from the sub-slits interfere when they arrive on a screen ...

- If path difference between wavelets from the edges is \( a \sin \theta = p \lambda \), \( p = 1, 2, \ldots \)
- the wavelets interfere destructively in pairs: wavelets 1 and 7 cancel (they have \( p \lambda / 2 \) difference!), and so do pairs 2 and 8, 3 and 9, …, 6 and 12!
- Diffraction pattern: central bright maximum, with minima and secondary maxima on either side …

![Diagram showing wavelets and path difference](image)

- size of \( a \) is exaggerated!
Summary: Diffraction

Central bright maximum;

**DARK** fringes for: \( a \sin \theta_p = p\lambda \), \( p = \pm 1, \pm 2, \pm 3, \ldots \)

\[
\sin \theta_p \approx \theta_p = p \frac{\lambda}{a}, \quad p = \pm 1, \pm 2, \pm 3, \ldots
\]

**position of dark fringes on screen:**

\[
\theta_p = \sin \theta_p = \tan \theta_p = \frac{y_p}{L} = p \frac{\lambda}{a}
\]

\[\Rightarrow y_p = p \frac{\lambda}{a} L\]

**width of the CENTRAL maximum:**

(distance between dark fringes on either side of the central maximum):

\[\Delta y_1 = 2 \frac{\lambda}{a} L\]

**Diffraction occurs always** near edges, holes, etc., and sets a fundamental limit to resolution.
A slot of width \( a = 5 \ \mu m \), illuminated by a \( \lambda = 0.500 \ \mu m \) point-like light source, and imaged on a screen 1.0 m behind the wire, will produce an image with a width of \( \ldots \) m.

\[ \Delta y_1 = 2 \frac{\lambda}{a} L \]
A monochromatic X-ray beam gives diffraction pattern when scattered off a crystal lattice; - as expected for WAVES but a beam of mono-energetic particles also gives a diffraction pattern!!

→ particles ↔ waves ?!
Electron Interference

A beam of “monochromatic” electrons “shining” on a double slit, produces an interference pattern!

• EVEN if they come 1 electron per second !!

BUT: ONLY if we do not measure WHICH of the two slits they go through!

- Aahh, the magic of quantum physics!
- deduced from the size $d$ of the slit, the momentum $p=mv$ of the electrons, and the interference pattern:

$$d \sin \theta_m = n\lambda; \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

$$\lambda = \frac{h}{p}$$

$$hc = 1240 \text{ eV} \cdot \text{nm}$$

remember: $1 \text{ eV} = 1 \text{ e J/C} = 1.6 \times 10^{-19} \text{ J}$
What is “Waving” for a Particle ??

A lot of discussion arose about the question of what exactly is “waving” in a “particle wave”;
- in an EM wave it is the $E$ and $B$ field vectors that are oscillating ...

how to interpret the wave function associated with particles ?
- A point-like particle at position $x$ represented by a wave ??

Try: a localized particle represented by a “wavelet” $\psi(x)$:

Try: a localized particle represented by a “wavelet” $\psi(x)$:

\[ \Delta \lambda \approx \lambda / n_{\text{waves}} \]

- exact wavelength is uncertain: \( \Delta \lambda \approx \lambda / n_{\text{waves}} \)
- thus, momentum has uncertainty:
\[ \Delta p_x = \Delta \left( \frac{h}{\lambda} \right) = \frac{h}{\lambda^2} \Delta \lambda = \frac{h}{\lambda n_{\text{waves}}} \]
- more waves (longer wavelet): \( \Delta x \nearrow \) and \( \Delta p_x \searrow \)

\[ \Delta x \Delta p_x \approx n_{\text{waves}} \lambda \times h/(n_{\text{waves}} \lambda) = h \]
- in fact, this is a minimum: Heisenberg’s Uncertainty Relationship:

\[ \Delta x \Delta p_x \geq \frac{h}{4\pi} \]